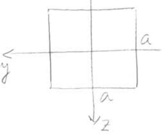
ОТПОРНОСТ МАТЕРИЈАЛА 1

Припрема за колоквијуме

Генерација 08

Моменти инерције од тежишних оса — I део



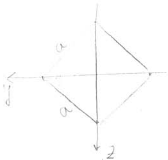
АКСИЈАЛНИ

$$I_y = I_z = \frac{a^4}{12}$$

ЦЕНТРИФУГАЛНИ

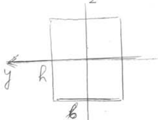
$$I_{yz} = 0$$

Ненад Јовановић



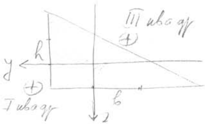
$$I_y = I_z = \frac{a^4}{12}$$

$$I_{yz} = 0$$



$$I_y = \frac{bh^3}{12} \quad I_z = \frac{hb^3}{12}$$

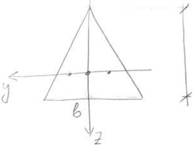
$$I_{yz} = 0$$



$$I_y = \frac{bh^3}{36} \quad I_z = \frac{hb^3}{36}$$

$$I_{yz} = \pm \frac{b^2h^2}{72}$$

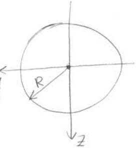
$$I_{yz} = -\frac{b^2h^2}{72}$$



$$I_y = \frac{bh^3}{36}$$

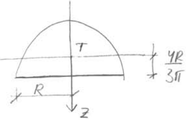
$$I_z = \frac{hb^3}{48}$$

$$I_{yz} = 0$$



$$I_y = I_z = \frac{R^4\pi}{4}$$

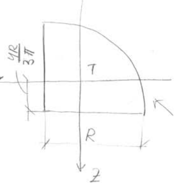
$$I_{yz} = 0$$



$$I_y = 0{,}1097\,R^4$$

$$I_z = \frac{R^4\pi}{8}$$

$$I_{yz} = 0$$

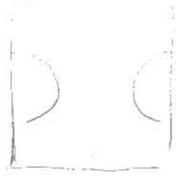


$$I_y = I_z = 0{,}0549\,R^4$$

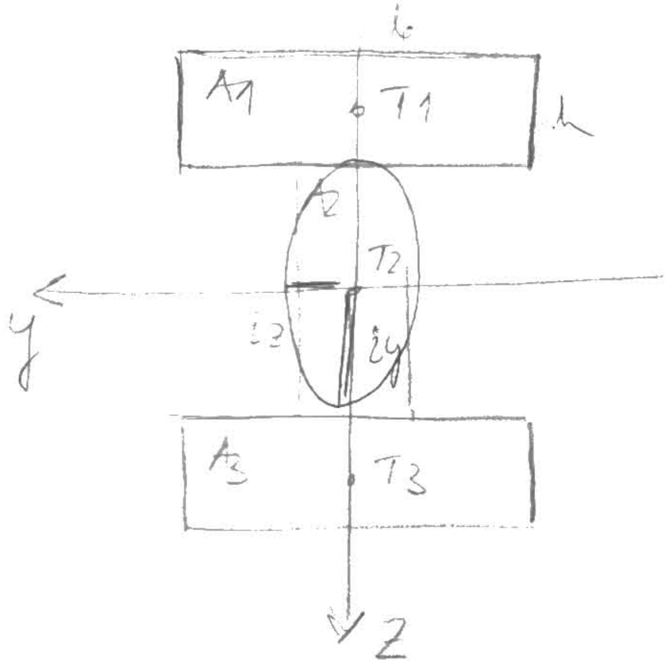
$$I_{yz} = \pm 0{,}01647\,R^4$$

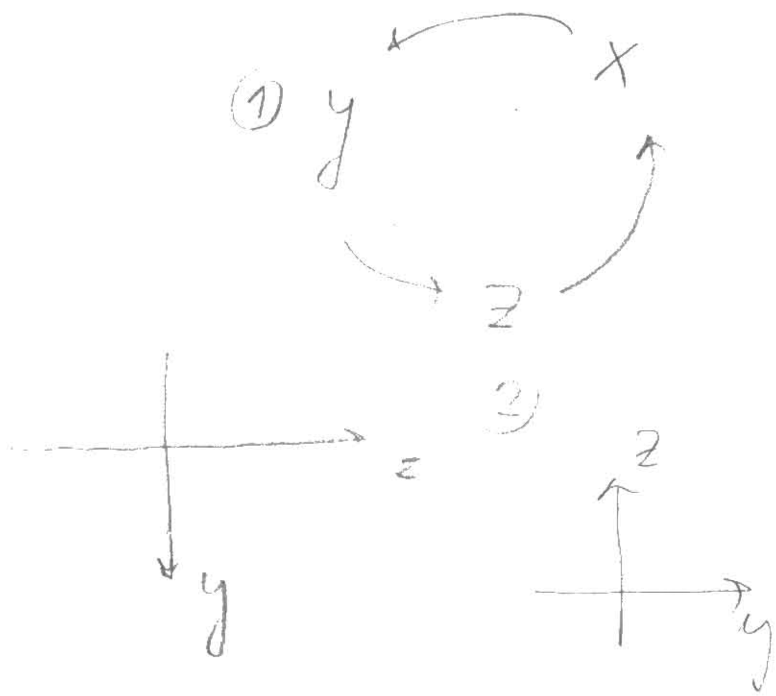
$$I_{yz} = -0{,}01647\,R^4$$

Моменти инерције



1. Пресеци са две осе симетрије

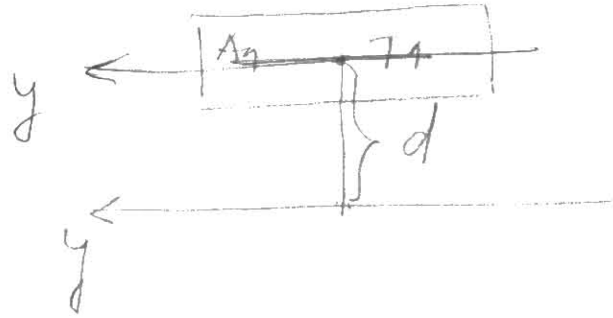




$A_1 = \quad T_1(y_1; z_1)$

$A_2 = \quad T_2(y_2; z_2)$

$A_3 = \quad T_3(y_3; z_3)$



$(+)\ I_y = I_{y1} + I_{y2} + I_{y3}$

$$I_{y1} = I_{y\,\text{сопст}} + I_{y\,\text{полож}} = \frac{bh^3}{12} + A_1 d_{1y}^2 = \frac{bh^3}{12} + A_1 (z_1)^2 \ cm^4$$

$(+)\ I_z = I_{z1} + I_{z2} + I_{z3}$

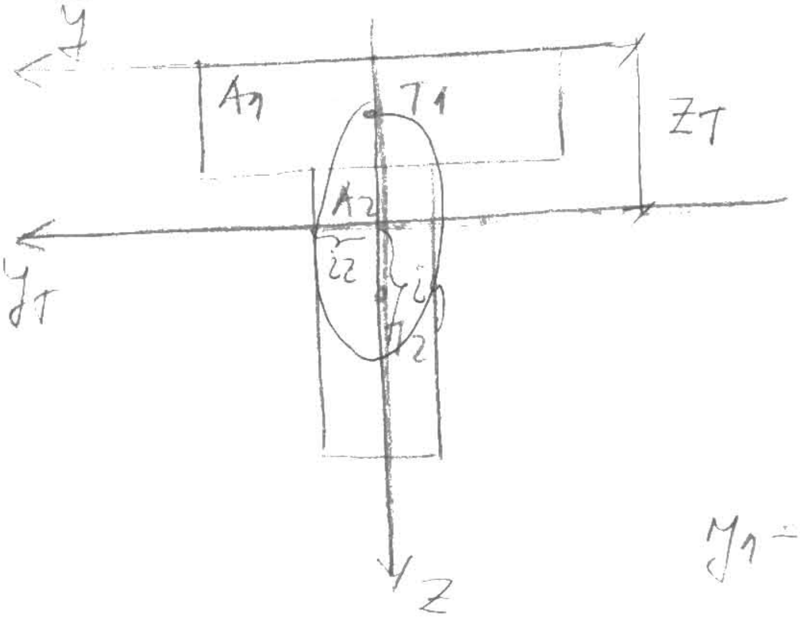
$$I_{z1} = I_{z\,\text{сопст}} + I_{z\,\text{полож}} = \frac{hb^3}{12} + A_1 (y_1)^2$$

$\boxed{I_{yz} = 0} \Rightarrow$ Главне осе су исто што и тежишне

Елипса инерције

$i_y = \sqrt{\frac{I_y}{A}} \qquad i_z = \sqrt{\frac{I_z}{A}}$

2. Пресек са једном осом симетрије



$$y_1 = \bar{y}_1 - y_T \qquad z_1 = \bar{z}_1 - z_T$$

$$A_1 = \quad \cancel{\bar{T}_1(\bar{y}_1, \bar{z}_1)} \quad T_1(y_1, z_1)$$

$$A_2 = \quad \cancel{\bar{T}_2(\bar{y}_2, \bar{z}_2)} \quad T_2(y_2, z_2)$$

$$z_T = \frac{\sum (A_i \cdot \bar{z}_i)}{\sum A_i} = \frac{A_1 \cdot \bar{z}_1 + A_2 \cdot \bar{z}_2}{A_1 + A_2}$$

$$I_y = I_{y_1} + I_{y_2} = \underbrace{\frac{bh^3}{12} + A_1 \cdot z_1^2} + \underbrace{\frac{bh^3}{12} + A_2 \cdot z_2^2}$$

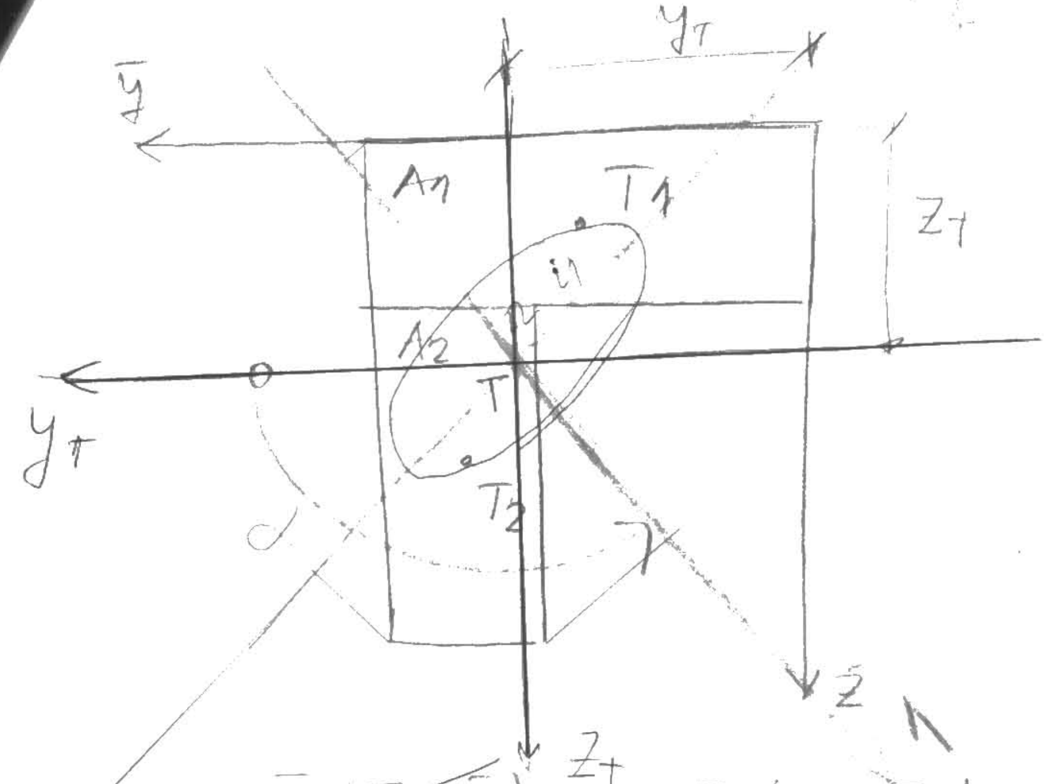
$$I_z = I_{z_1} + I_{z_2} = \underbrace{\frac{hb^3}{12} + A_1 y_1^2} + \underbrace{\frac{hb^3}{12} + A_2 \cdot y_2^2}$$

$\boxed{I_{yz} = 0} \Rightarrow$ Главне осе су исто што и тежишне

Елипса инерције

$$i_y = \sqrt{\frac{I_y}{A}} \qquad i_z = \sqrt{\frac{I_z}{A}}$$

3. Пресек без осе симетрије



$A_1 =$ ~~$\bar T_1(\bar y_1; \bar z_1)$~~ $\quad T_1(y_1; z_1)$

$A_2 =$ ~~$\bar T_2(\bar y_2; \bar z_2)$~~ $\quad T_2(y_2; z_2)$

$$y_T = \frac{\sum A_i \bar y_i}{A} = \frac{A_1 \bar y_1 + A_2 \bar y_2}{A_1 + A_2} \qquad z_T = \frac{A_1 \bar z_1 + A_2 \bar z_2}{A_1 + A_2}$$

Моменти инерције око тежишних оса

$$\oplus \; I_y = I_{y1} + I_{y2} = \underbrace{\frac{bh^3}{12} + A_1 z_1^2} + \ldots$$

$$\oplus \; I_z = I_{z1} + I_{z2} = \underbrace{\frac{hb^3}{12} + A_1 y_1^2} + \ldots\ldots$$

$$\pm \; I_{yz} = I_{yz1} + I_{yz2} = \quad \neq 0$$

знак!

$$I_{yz1} = I_{yz\,\text{соп}} + I_{yz\,\text{пол}} = 0 + A_1 \cdot y_1 \cdot z_1$$

Моменти инерције око главних оса

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

$\oplus$ $I_1 = max$

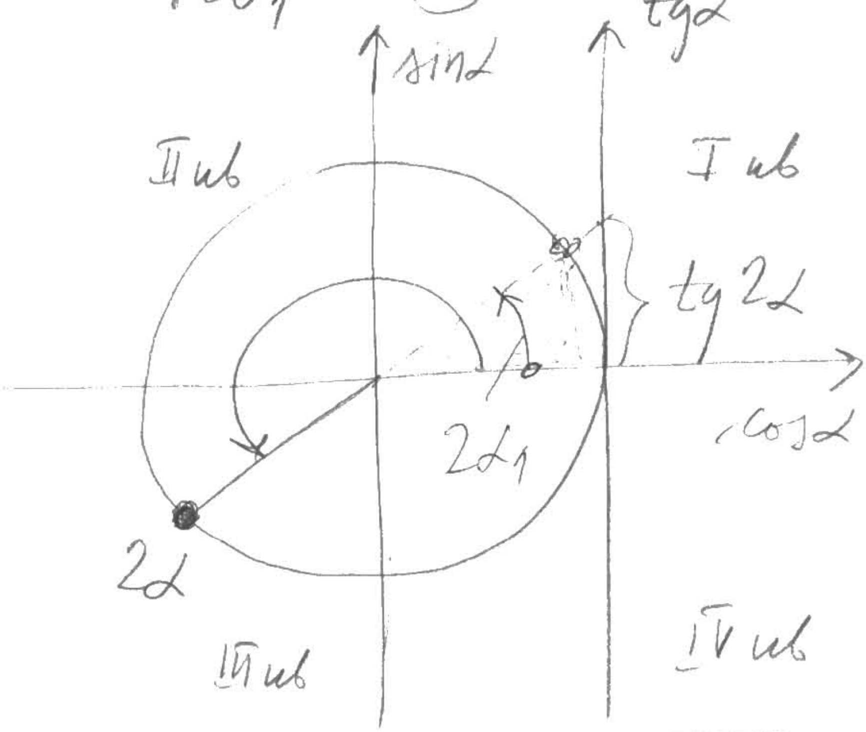
$\ominus$ $I_2 = min$

Положај главних оса

$$tg\, 2\alpha_1 = \frac{-2 I_{yz}}{I_y - I_z} = \frac{\sin 2\alpha_1}{\cos 2\alpha_1} = \frac{\ominus}{\ominus}$$

III квадрант

$2\alpha = 180° + |2\alpha_1|$



I квадрант $\frac{+}{+}$

$2\alpha = 2\alpha_1$

II квадрант $\frac{+}{-}$

$2\alpha = 180° - |2\alpha_1|$

III квадрант $\frac{-}{-}$

$2\alpha = 180° + |2\alpha_1|$

IV квадрант $\frac{-}{+}$

$2\alpha = 360° - |2\alpha_1|$

$tg\, 2\alpha_1 = \frac{+}{0} = +\infty$

$2\alpha = 90 \quad \alpha = 45°$

$tg\, 2\alpha_1 = \frac{-}{0} = -\infty$

$2\alpha = 270° \quad \alpha = 135°$

Елипса инерције

$$i_1 = \sqrt{\frac{I_1}{A}} \qquad i_2 = \sqrt{\frac{I_2}{A}}$$

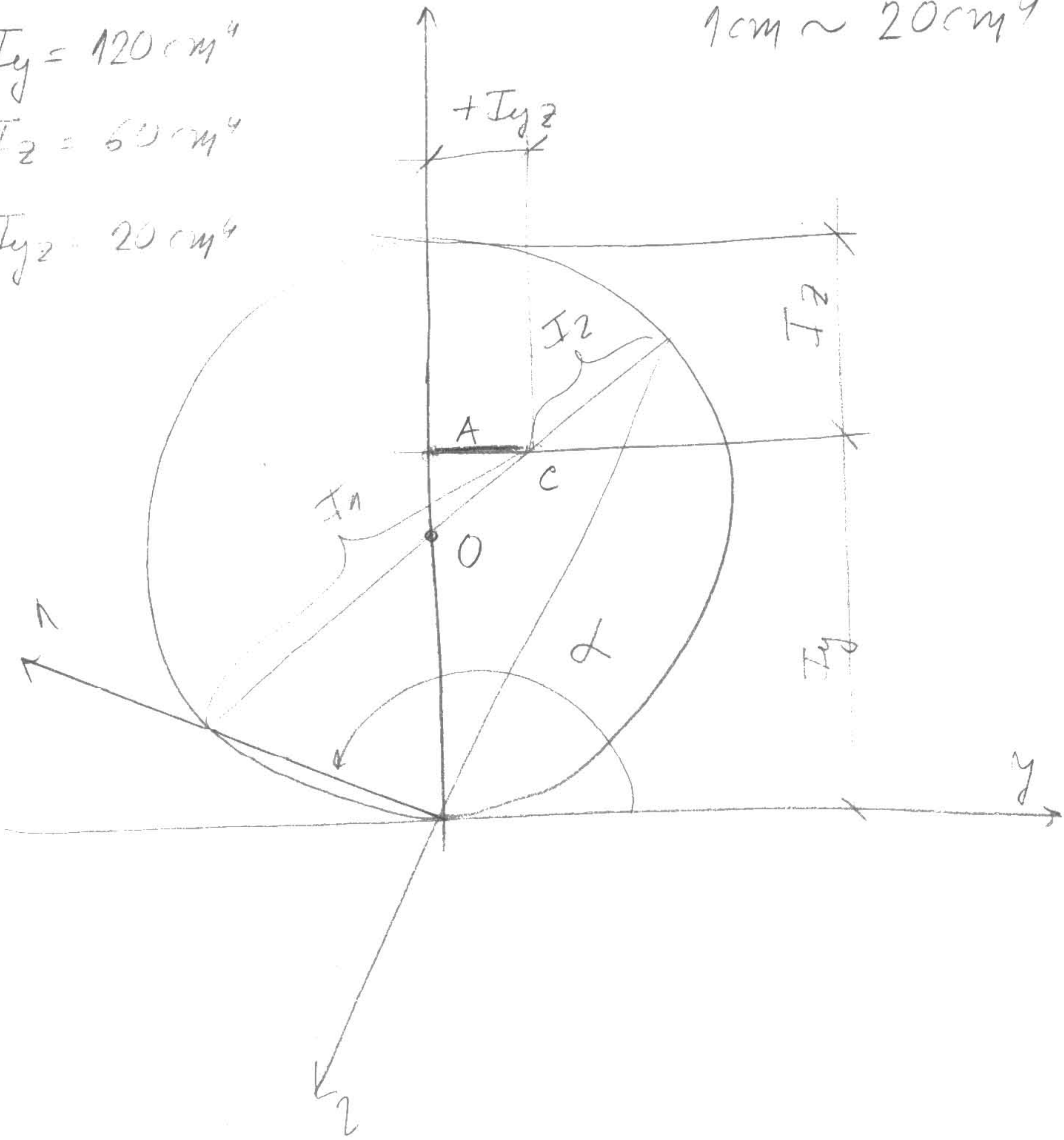
Моров круг инерције

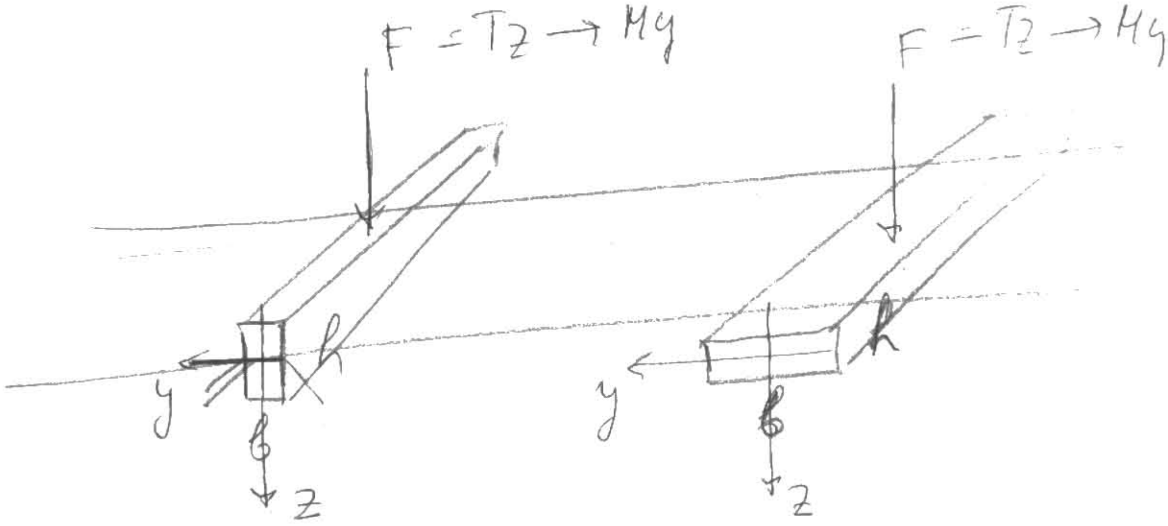
$I_y = 120\,cm^4$

$I_z = 60\,cm^4$

$I_{yz} = 20\,cm^4$

$1\,cm \sim 20\,cm^4$





$$I_y = \frac{bh^3}{12} \gg I_y = \frac{bh^3}{12}$$

$$\sigma_x = \frac{M_y}{I_y} z$$

1. Задатак

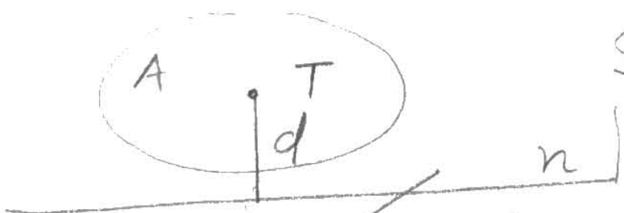
$A_1 = 6 \quad T_1(3,99;\ 1,7)$

$A_2 = 4,5 \quad T_2(3,49;\ 3,7)$

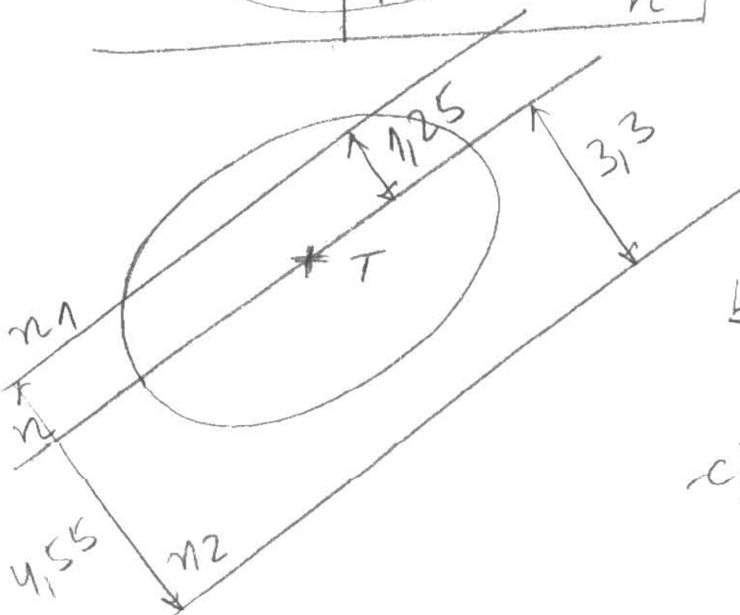
$$y_T = \frac{6\cdot 3,99 + 4,5\cdot 3,49}{10,5} = \text{ cm}$$

$$z_T = \frac{6\cdot 1,7 + 4,5\cdot 3,7}{10,5} \text{ cm}$$

2. S = статички момент површине



$S_n = A\cdot d \ \text{cm}^3$



a) $S_{n1} = 85\text{cm}^3 \quad S_{n2} = ?$

$S_{n1} = A\cdot 1,25$

$S_{n2} = A\cdot 3,3$

b) $S_{n1} = 85\text{cm}^3 \quad A = ?$

$S_{n1} = A\cdot 1,25$

c) $S_{n1} = 85\,\text{cm}^3 \quad S_n = ?$

$S_n = 0$

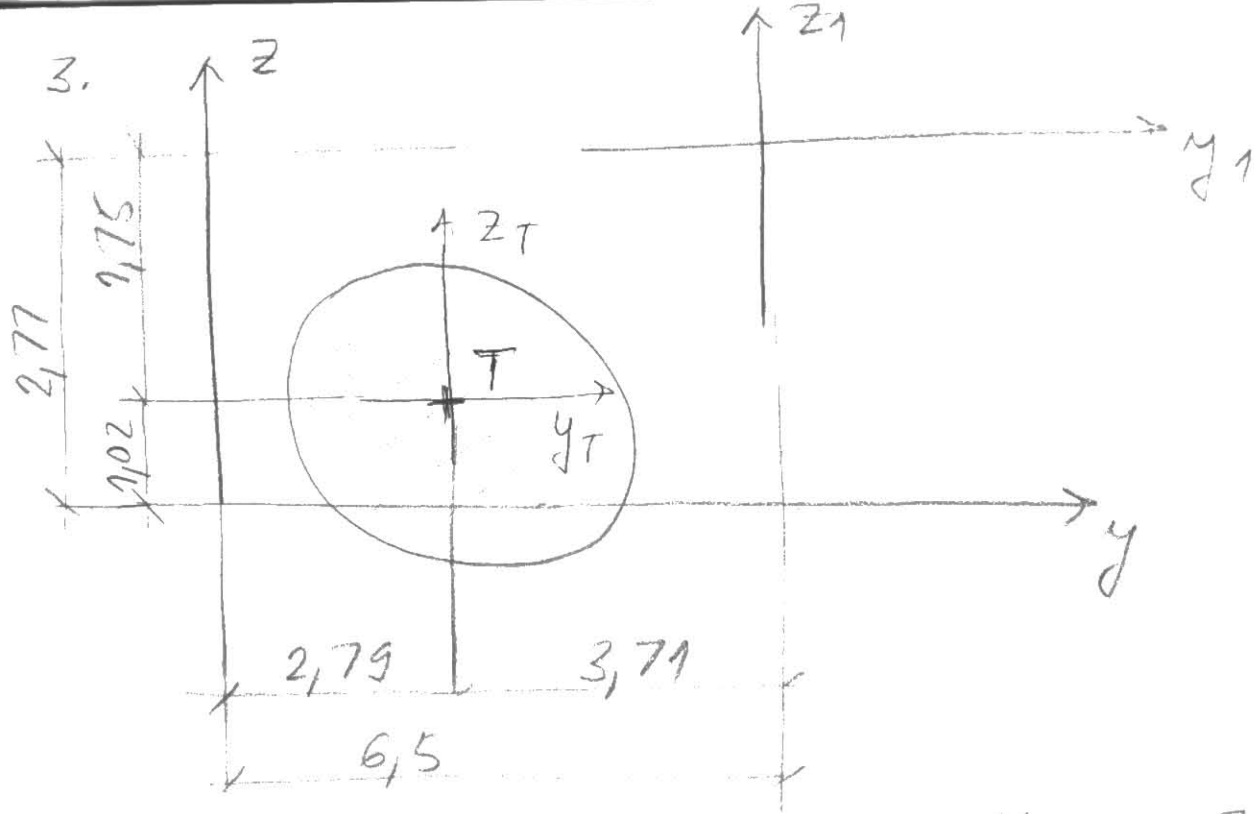
d) $I_{n1} = 200\text{cm}^4 \quad A = 50\text{cm}^2 \quad I_{n2} = ?$

$I_{n1} = I_n + 50\cdot(1,25)^2$

$I_{n2} = I_n + 50(3,3)^2$

e) $I_{n1} = 200\ \text{cm}^4 \quad A = 50\text{cm}^2 \quad I_n = ?$

$I_{n1} = I_n + 50(1,25)^2$



$\overset{y}{}\overset{z}{}$ $T_{yz}(2{,}79;\ 1{,}02)$ $\qquad$ $T_{y_1 z_1}(-3{,}71;\ -1{,}75)$

a) $A = 50 \quad I_{z_1} = 550 \quad I_z = ?$

$I_{z_1} = I_{z_T} + 50 \cdot (-3{,}71)^2$

$I_z = I_{z_T} + 50\ (2{,}79)^2$

b) $A = 50 \quad I_z = 400 \quad I_{z_1} = ?$

$I_z = I_{z_T} + 50 \cdot (2{,}79)^2$

$I_{z_1} = I_{z_T} + 50\ (-3{,}71)^2$

c) $A = 50 \quad I_{y_1 z_1} = 250 \quad I_{yz}$

$I_{y_1 z_1} = I_{y_T z_T} + 50\,(-3{,}71)(-1{,}75)$

$I_{yz} = I_{y_T z_T} + 50\,(3{,}79)(1{,}02)$

d) $S_y = 80\,cm^3 \quad S_z = ?$

$S_y = A \cdot 1{,}02$

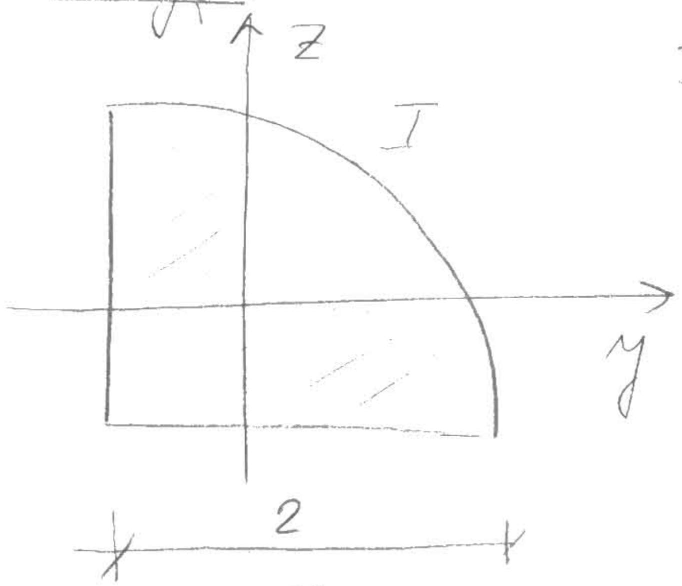
$S_z = A \cdot 2{,}79$

e) $A = 50 \quad I_{y_1} = 350 \quad I_y = ?$

$I_{y_1} = I_{y_T} + 50 \cdot (-1{,}75)^2$

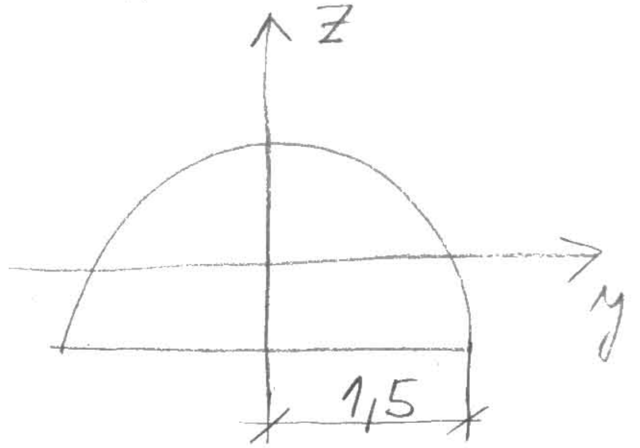
$I_y = I_{y_T} + 50\ (1{,}02)^2$

4. Задачи



$I_y = I_z = 0{,}0549\,(2)^4\ cm^4$

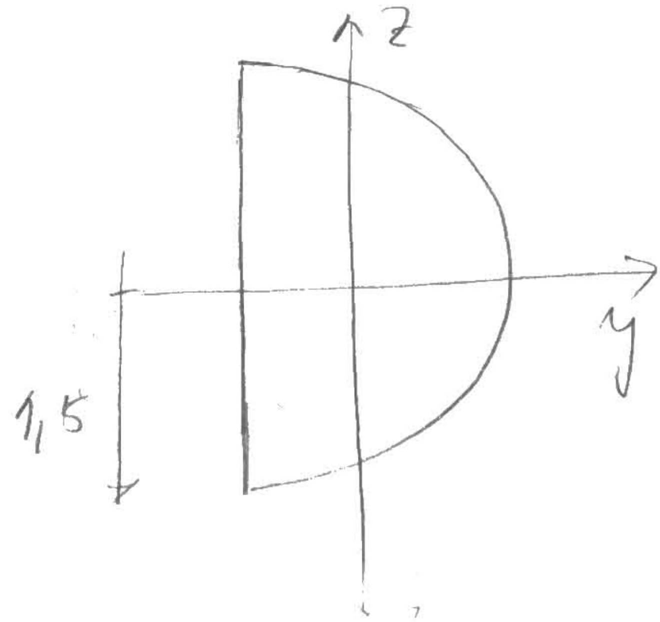
$I_{yz} = -0{,}01647\,(2)^4\ cm^4$



$I_y = 0{,}1097\,(1{,}5)^4$

$I_z = \dfrac{(1{,}5)^4\pi}{8}$

$I_{yz} = 0$

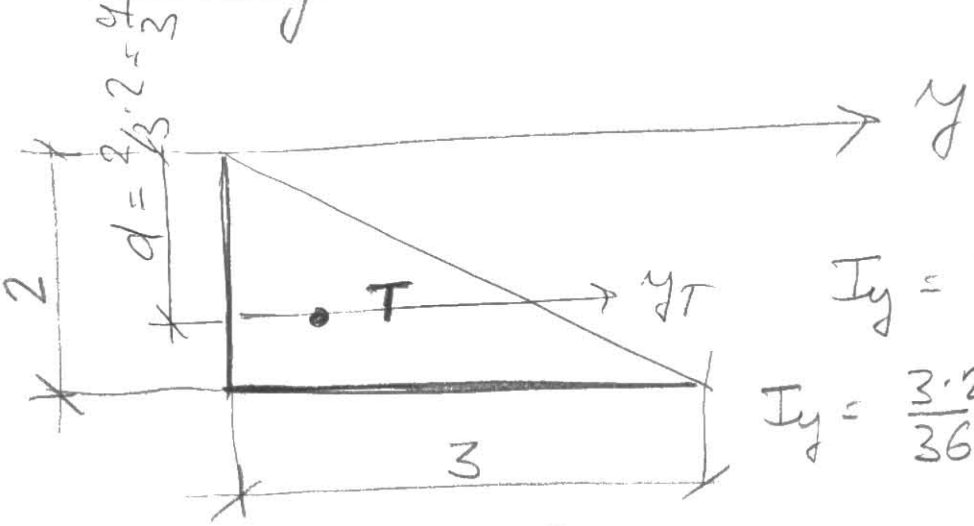


$I_y = \dfrac{(1{,}5)^4\pi}{8}$

$I_z = 0{,}1097\,(1{,}5)^4\ cm^4$

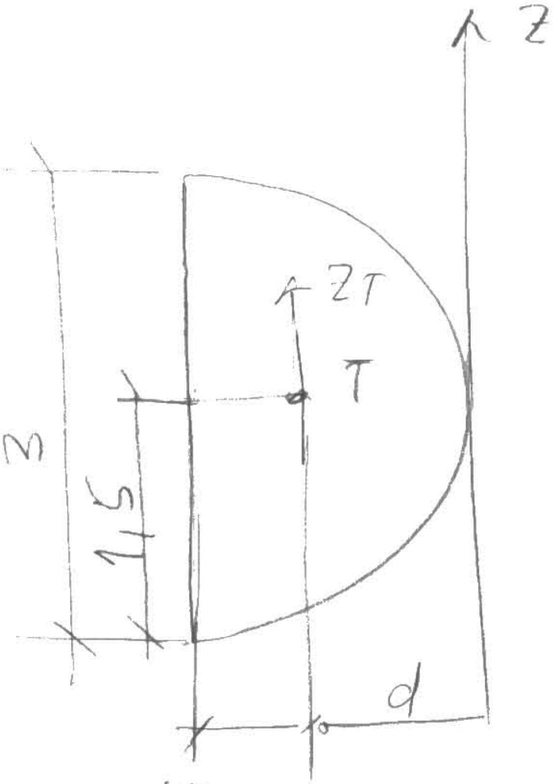
$I_{yz} = 0$

5 - задачи



$I_y = I_{y_T} + A\cdot d^2$

$I_y = \frac{3\cdot 2^3}{36} + \left(\frac{1}{2}\cdot 3\cdot 2\right)\cdot\left(\frac{4}{3}\right)^2 \text{ cm}^4$



$\frac{4R}{3\pi} = \frac{4\cdot 1,5}{3\pi} = \frac{6}{3\pi} = \frac{2}{\pi}$

$d = R - \frac{4R}{3\pi} = \left(1,5 - \frac{2}{\pi}\right)$

$I_z = I_z + A\cdot d^2 = 0,1097\cdot(1,5)^4 + \frac{1}{2}(1,5)^2\pi\cdot\left(1,5 - \frac{2}{\pi}\right)^2$

6.

	I_{y_T}	I_{z_T}	I_{yz}	A
1	100	170	0	20
1	80	150	25	18

$T_1(-0{,}86;\ -0{,}29)$ (y; z)

$T_2(1{,}14;\ 0{,}38)$

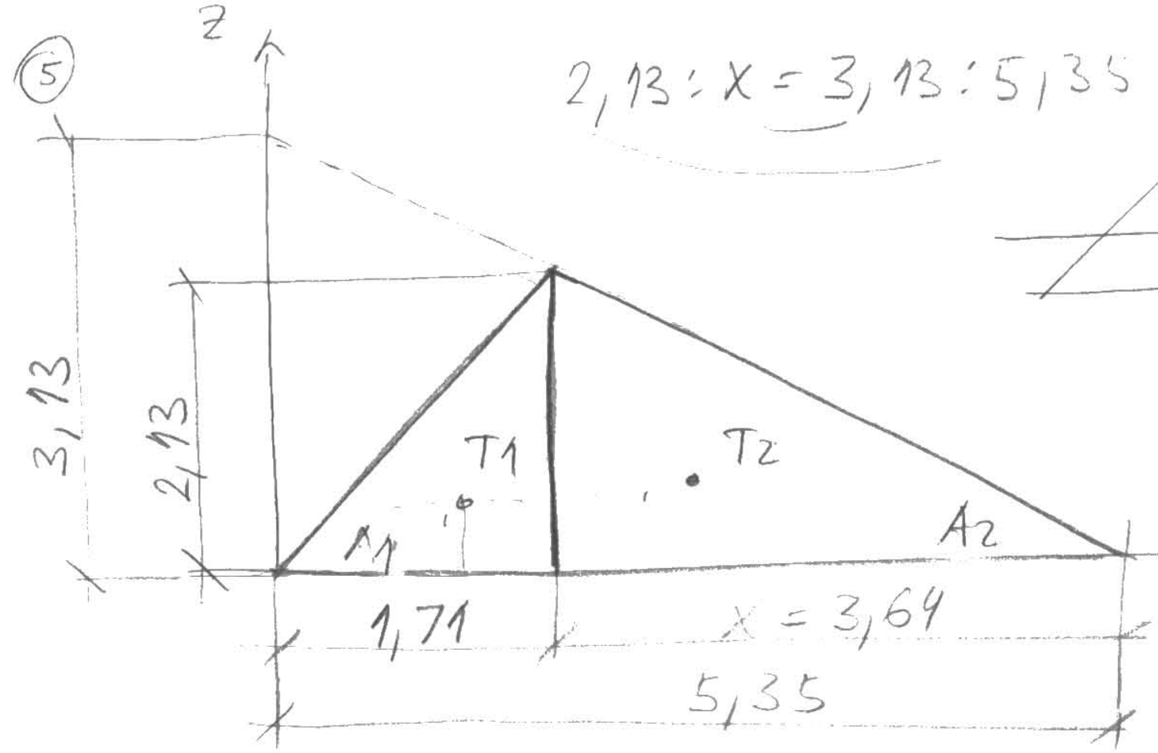
$I_y = 100 + 20\cdot(-0{,}29)^2 + 80 + 18\cdot(0{,}38)^2$

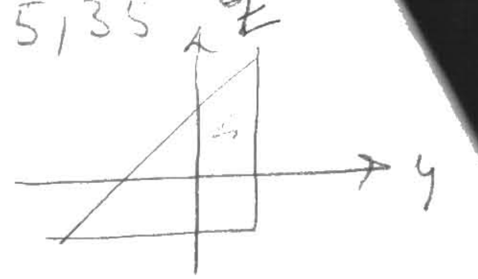
$I_z = 170 + 20\cdot(-0{,}86)^2 + 150 + 18\,(1{,}14)^2$ cm^4

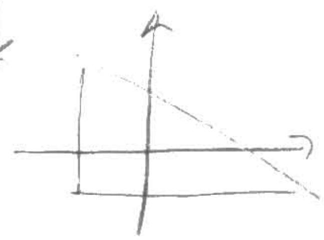
$I_{yz} = 0 + 20(-0{,}86)(-0{,}29) + 25 + 18(1{,}14)(0{,}38)$ cm^4

5

$$2{,}13 : x = 3{,}13 : 5{,}35$$







$$A_1 = \frac{1}{2} \cdot 1{,}71 \cdot 2{,}13 = 1{,}82 \qquad T_1\,(1{,}14\,;\,0{,}71)$$

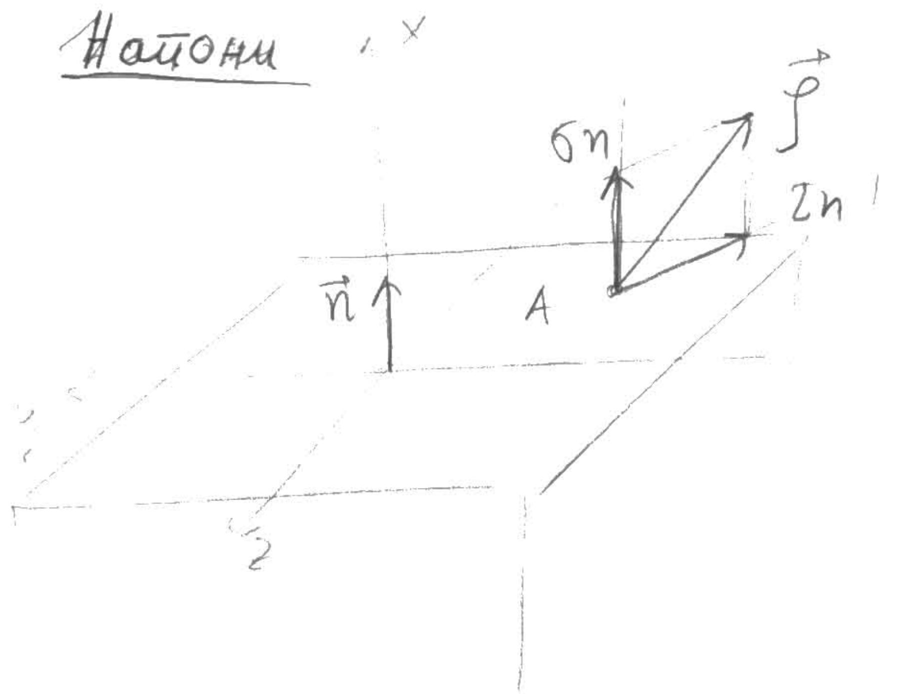
$$A_2 = \frac{1}{2} \cdot 3{,}64 \cdot 2{,}13 = 3{,}88 \qquad T_2\,(2{,}92\,;\,0{,}71)$$

$$I_y = \frac{1{,}71 \cdot 2{,}13^3}{36} + 1{,}82 \cdot (0{,}71)^2 + \frac{3{,}64 \cdot 2{,}13^3}{36} + 3{,}88 \cdot (0{,}71)^2 \ \mathrm{cm}^4$$

$$I_z = \frac{2{,}13 \cdot 1{,}71^3}{36} + 1{,}82 \cdot (1{,}14)^2 + \frac{2{,}13 \cdot 3{,}64^3}{36} + 3{,}88\,(2{,}92)^2 \ \mathrm{cm}^4$$

$$I_{yz} = \frac{1{,}71^2 \cdot 2{,}13^2}{72} + 1{,}82 \cdot 1{,}14 \cdot 0{,}71 - \frac{3{,}64^2 \cdot 2{,}13^2}{72} + 3{,}88 \cdot 2{,}92 \cdot 0{,}71 \ \mathrm{cm}^4$$

Напони



1\. $\vec{p}$ = тотален напон $\vec{\sigma}_n$ = нормална компонентални комп напон

$\vec{p} = \vec{\sigma}_n + \vec{\tau}_n$ $\vec{\tau}_n$ = смичувачка комп напон

$p^{(n)} = \sqrt{\sigma_n^2 + \tau_n^2}$

a) $\vec{p}^{(n)} = (-5\vec{i} + 8\vec{j} + 5\vec{k})\,MPa$ $\vec{n} = 0{,}8\vec{j} + 0{,}6\vec{k}$

$\sigma_n = \vec{p}^{(n)} \cdot \vec{n} = 8 \cdot 0{,}8 + 5 \cdot 0{,}6 = 9{,}4\,MPa$

$\vec{\sigma}_n = \sigma_n \cdot \vec{n} = (9{,}4 \cdot 0{,}8)\vec{j} + (9{,}4 \cdot 0{,}6)\vec{k} = \underline{(7{,}52\vec{j} + 5{,}64\vec{k})\,MPa}$

$\vec{\tau}_n = \vec{p}^{(n)} - \vec{\sigma}_n = \underline{-5\vec{i} + 0{,}48\vec{j} - 0{,}64\vec{k}}$

2\. $\vec{m} = \{\cos\alpha;\ \cos\beta;\ \cos\gamma\} \Rightarrow$ јединичен вектор за $\vec{\tau}_n$

$|\vec{\tau}_n| = \sqrt{5^2 + 0{,}48^2 + 0{,}64^2} = 5{,}06\,MPa$

$\cos\alpha = \frac{\tau_{nx}}{|\vec{\tau}_n|} = \frac{-5}{5{,}06}$ $\cos\beta = \frac{0{,}48}{5{,}06}$ $\cos\gamma = \frac{-0{,}64}{5{,}06}$

3.4.

$$S = \begin{array}{c} \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \end{array} \\ \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix} \end{array} MPa \quad \begin{array}{l} \to p_x \\ \to p_y \\ \to p_z \end{array}$$

$$\vec{n} = \frac{4}{\sqrt{17}}\vec{i} + \frac{1}{\sqrt{17}}\vec{j}$$

$$p_x = 5 \cdot \frac{4}{\sqrt{17}} + 2 \cdot \frac{1}{\sqrt{17}} = \frac{22}{\sqrt{17}} \ MPa$$

$$p_y = 2 \cdot \frac{4}{\sqrt{17}} + 3 \cdot \frac{1}{\sqrt{17}} = \frac{11}{\sqrt{17}} \ MPa$$

$$p_z = 4 \cdot \frac{1}{\sqrt{17}} = \frac{4}{\sqrt{17}} \ MPa$$

$$\vec{p} = \left\{ \frac{22}{\sqrt{17}} ; \frac{11}{\sqrt{17}} ; \frac{4}{\sqrt{17}} \right\} MPa$$

$$\vec{p} = \frac{22}{\sqrt{17}}\vec{i} +$$

5.8.

$$\vec{p}^{(m)} = 2\vec{i} + 3\vec{j} + \vec{k} \qquad \vec{m} = 0{,}8\vec{i} + 0{,}6\vec{k}$$

$$\vec{n}_1 = 0{,}8\vec{i} + 0{,}6\vec{j}$$

$\sigma_1 = ?$

$$\boxed{\vec{p}^{(m)} \cdot \vec{n}_1 = \vec{\sigma}_1 \cdot \vec{m}} \qquad \boxed{\vec{\sigma}_1 = \sigma_1 \cdot \vec{n}_1}$$

$$\vec{p}^{(m)} \cdot \vec{n}_1 = \sigma_1 \cdot \vec{n}_1 \cdot \vec{m}$$

$$2 \cdot 0{,}8 + 3 \cdot 0{,}6 = \sigma_1 \cdot (0{,}8 \cdot 0{,}8) \qquad \underline{\sigma_1 = 5{,}31 \ MPa}$$

$$\vec{p}^{(m)} \cdot \vec{n}_2 = \vec{\sigma}_2 \cdot \vec{m} \implies \sigma_2$$

$$\vec{p}^{(m)} \cdot \vec{n}_3 = \vec{\sigma}_3 \cdot \vec{m}$$

5.7.

$$S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix} MPa \qquad S = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z = 8$$

$$I_2 = \begin{vmatrix} 3 & 4 \\ 4 & 0 \end{vmatrix} + \begin{vmatrix} 5 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 5 & 2 \\ 2 & 3 \end{vmatrix} = -16 + 11 = -5$$

$$I_3 = \det S = 5(-16) - 2 \cdot 0 = -80$$

секуларна једначина

$$\sigma^3 - I_1 \cdot \sigma^2 + I_2 \cdot \sigma - I_3 = 0$$

$$\sigma^3 - 8\sigma^2 - 5 \cdot \sigma + 80 = 0$$

7. $S = S^{сферно} + S^{дев}$

$$S^{сферно} = \begin{bmatrix} \sigma_s & 0 & 0 \\ 0 & \sigma_s & 0 \\ 0 & 0 & \sigma_s \end{bmatrix}$$

$$\sigma_s = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{8}{3}$$

$S^{девијаторски}$

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} \end{bmatrix} + \begin{bmatrix} \frac{7}{3} & 2 & 0 \\ 2 & \frac{1}{3} & 4 \\ 0 & 4 & -\frac{8}{3} \end{bmatrix}$$

$$I_{2дев} = \begin{vmatrix} \frac{1}{3} & 4 \\ 4 & -\frac{8}{3} \end{vmatrix} + \begin{vmatrix} \frac{7}{3} & 0 \\ 0 & -\frac{8}{3} \end{vmatrix} + \begin{vmatrix} \frac{7}{3} & 2 \\ 2 & \frac{1}{3} \end{vmatrix} \ldots\ldots$$

<u>II страна</u>

8.
$$\begin{array}{cc} & \begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{array} \\ S = & \begin{bmatrix} 3x+3y & 2x & x+z^2 \\ 2x & 7 & 2+xy \\ x+z^2 & 2+yz & x+y \end{bmatrix} \cdot 10^{-3}\,Pa \end{array} \quad \begin{array}{l} +F_x = 0 \\ +F_y = 0 \\ +F_z = 0 \end{array}$$

$$10^{-3}(3+0+2z)+F_x=0 \qquad F_x=(-3-2z)\cdot 10^{-3}\,Pa$$

$$10^{-3}(2+0+0)+F_y=0 \qquad F_y=$$

$$10^{-3}(1+z+0)+F_z=0 \qquad F_z$$

$$F=\{F_x;F_y;F_z\}$$

---

9. равно стање напона $\det S=0$

$$S=\begin{bmatrix} 4 & 2 & 0 \\ 2 & A & 1 \\ 0 & 1 & -2 \end{bmatrix} MPa$$

$$\det S=0 \qquad 4(-2A-1)-2[2\cdot(-2)]=0$$

$$-8A-4+8=0 \qquad -8A+4=0 \qquad A=\frac{1}{2}$$

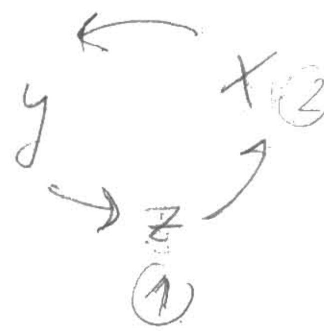
10.
$$S=\begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 1 \\ -1 & 1 & \frac{4}{3} \end{bmatrix} \qquad \begin{vmatrix} x & y & z \\ 1 & 0 & -1 \\ 0 & 3 & 1 \end{vmatrix}=0$$

$$3x-y\cdot(1)+z\cdot 3=0$$

$$3x-y+3z=0 \qquad \underline{\{3;-1;3\}}$$

11.

$$S = \begin{bmatrix} 5 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 2 \end{bmatrix} MPa$$



11.1.

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1/2} = \frac{2+5}{2} \pm \sqrt{\left(\frac{2-5}{2}\right)^2 + (-1)^2}$$

$$\sigma_{1/2} = 3{,}5 \pm 1{,}80 \qquad \underline{\sigma_1 = 5{,}3\,MPa} \qquad \underline{\sigma_2 = 1{,}7\,MPa}$$

$$tg\, 2\alpha_1 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\cdot(-1)}{2-5} = \frac{-2}{-3}$$

<u>III квадрант</u>

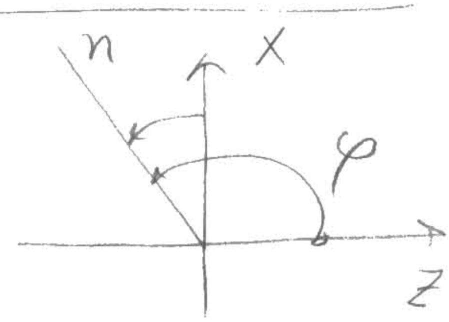
$$2\alpha = 180^\circ + |2\alpha_1| = 180 + 33{,}69$$

$$\underline{\alpha = 106{,}84^\circ}$$

11.2.

$$\tau_{max} = \frac{|\sigma_1 - \sigma_2|}{2} = \frac{|5{,}3 - (1{,}7)|}{2} = 1{,}8\,MPa$$

$$\alpha_{\tau max} = \alpha + 45^\circ = 151{,}84^\circ$$



11.3. $\varphi_z = 60^\circ$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\varphi + \tau \sin 2\varphi$$

$$\tau_n = \frac{\sigma_x - \sigma_y}{2}\sin 2\varphi - \tau \cos 2\varphi$$

$$\sigma_n = \frac{2+5}{2} + \frac{2-5}{2}\cos 120^\circ + (-1)\sin 120 = \underline{3{,}38\,MPa}$$

$$\tau_n = \frac{2-5}{2}\sin 120 - (-1)\cos 120 = \underline{-1{,}8\,MPa}$$

11.4. $\sigma_n = 3{,}38 \quad \tau_n = -1{,}8\ MPa$

$$p^{(n)} = \sqrt{\sigma_n^2 + \tau_n^2} = \sqrt{3{,}38^2 + (-1{,}8)^2} = 3{,}83\ MPa$$

11.5. $\tau_{max} = 1{,}8\ MPa$

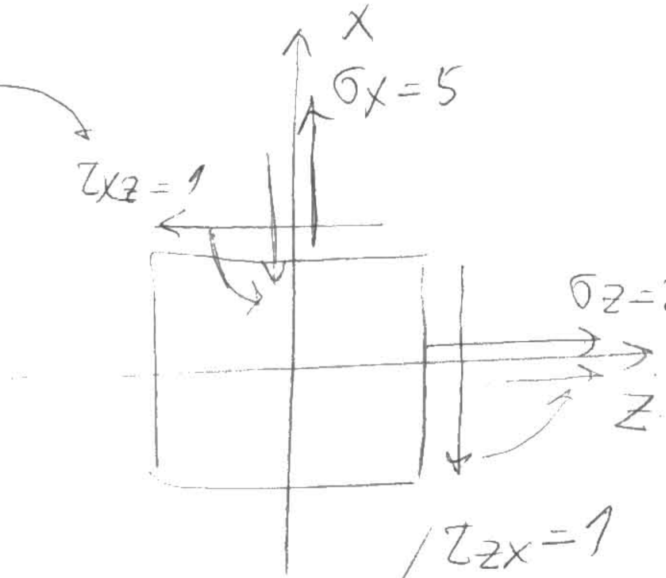
$$\sigma_{\tau max} = \frac{\sigma_1 + \sigma_2}{2} = \frac{5{,}3 + 1{,}7}{2} = 3{,}5\ MPa$$

$$p_{\tau max} = \sqrt{\sigma^2 + \tau^2} = \sqrt{3{,}5^2 + 1{,}8^2} = 3{,}94\ MPa$$

11.12. Моров круг

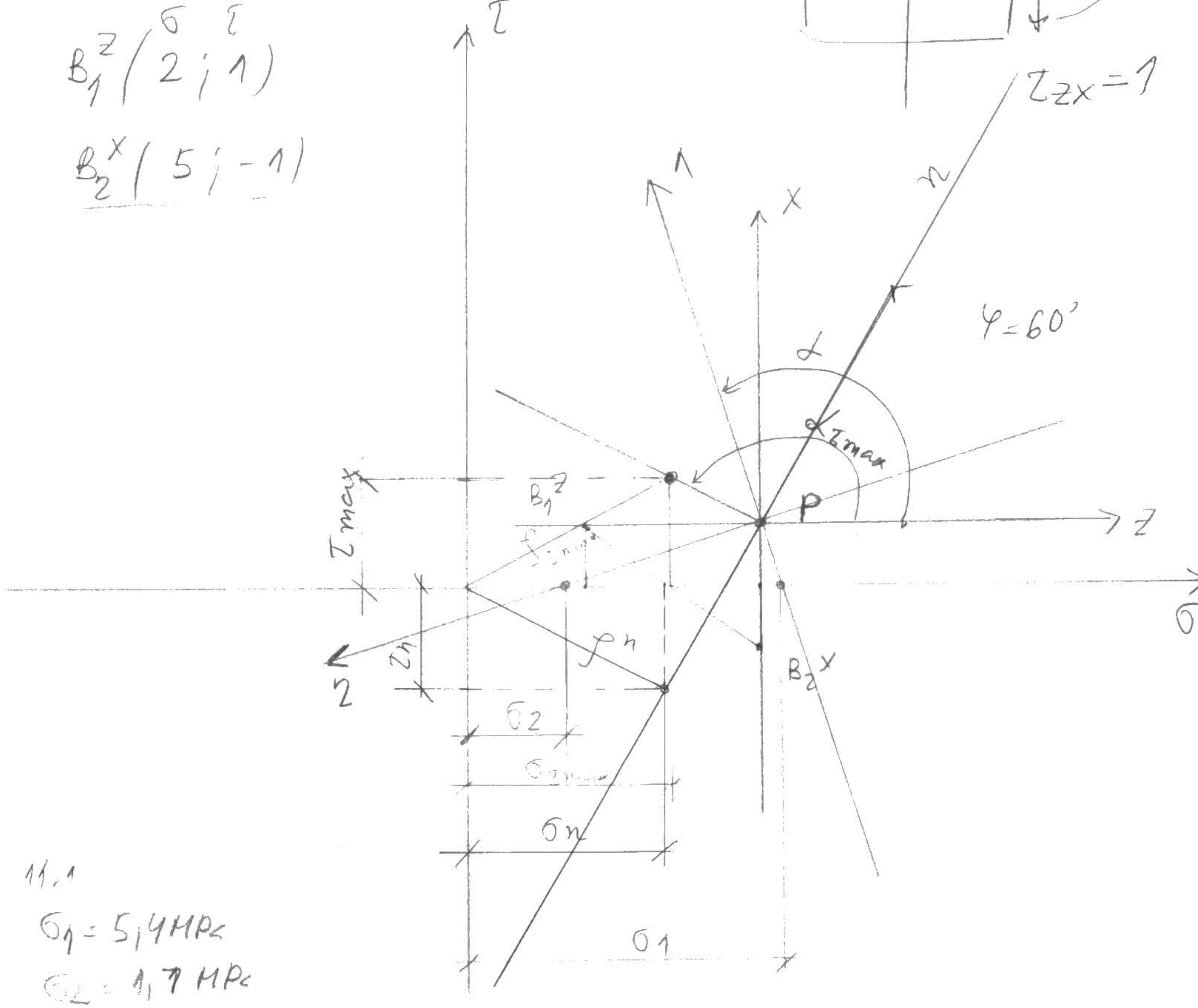
$$S = \begin{bmatrix} 5 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 2 \end{bmatrix} MPa$$

τ_{zx}



$B_1^Z(2;1)$ (σ, τ)

$B_2^X(5;-1)$



11.1
$\sigma_1 = 5,4$ MPa
$\sigma_2 = 1,7$ MPa
α

11.2 $\tau_{max} = 1,8$
$\alpha =$

11.3.

7) $I_y = 125 \quad I_z = 100 \quad I_{yz} = 30$ см^4

7

$$I_{1,2} = \frac{125+100}{2} \pm \sqrt{\left(\frac{125-100}{2}\right)^2 + 30^2}$$

$$tg\, 2\alpha_1 = \frac{-2\cdot 30}{125-100} = \frac{-60}{25}$$

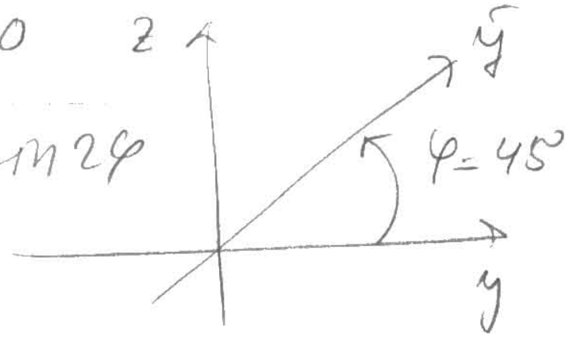
IV квадрант

$2\alpha = 360' - 67{,}38$

$\alpha = 146{,}31°$

8. $I_y = 125 \quad I_z = 100 \quad I_{yz} = 30$

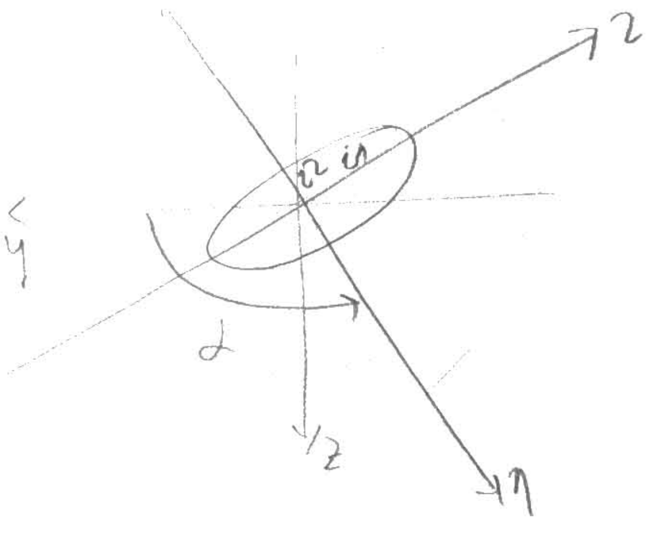
$$I_{\bar y} = \frac{1}{2}(I_y + I_z) + \frac{1}{2}(I_y - I_z)\cos 2\varphi - I_{yz}\sin 2\varphi$$



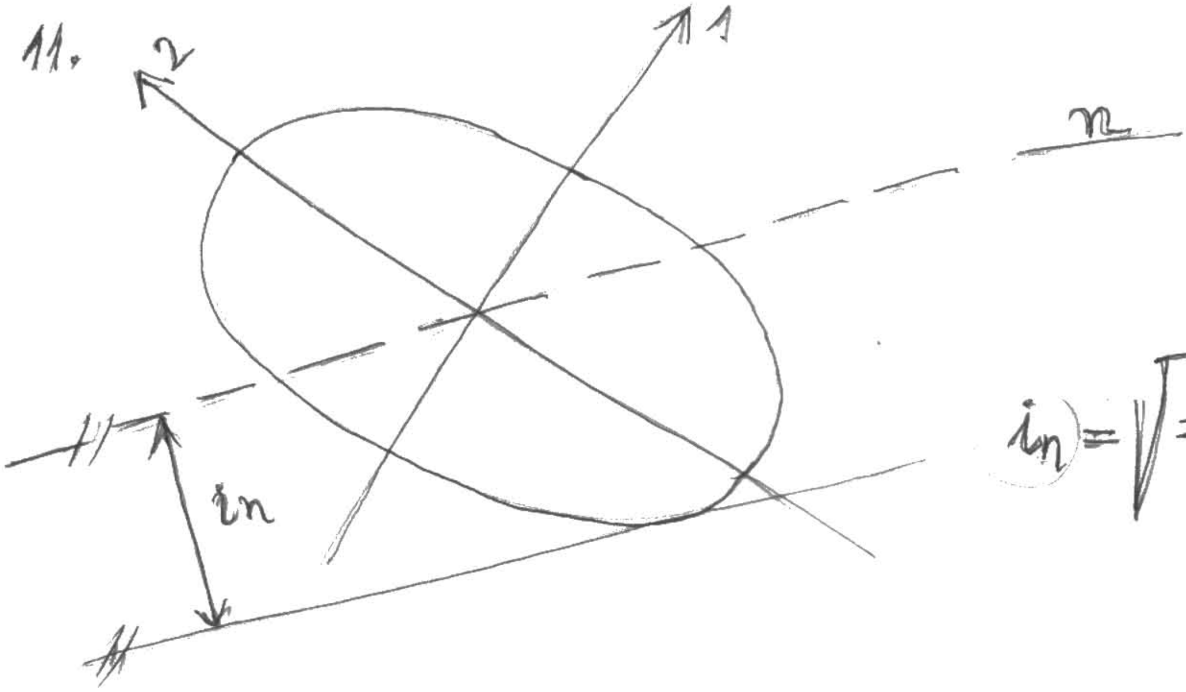
9. 1 cm = 20 cm^4

$I_1 \quad I_2$

10. $I_1 = \quad I_2 = \quad A =$ $\qquad i_1 = \sqrt{\frac{I_1}{A}} \qquad i_2 = \sqrt{\frac{I_2}{A}}$



11.



$$i_n = \sqrt{\frac{I_n}{A}}$$

Деформације

1. a)

$$D=\begin{bmatrix} \overset{\varepsilon_{11}}{2} & \overset{\varepsilon_{12}}{0} & \overset{\varepsilon_{13}}{3} \\ \overset{\varepsilon_{21}}{0} & \overset{\varepsilon_{22}}{1} & \overset{\varepsilon_{23}}{2} \\ \overset{\varepsilon_{31}}{3} & \overset{\varepsilon_{32}}{2} & \overset{\varepsilon_{33}}{0} \end{bmatrix} 10^{-6}$$

$$\vec{n}=\overset{(1)}{\frac{1}{3}}\vec{i}+\overset{(2)}{\frac{2}{3}}\vec{j}+\overset{(3)}{\frac{2}{3}}\vec{k}$$

$$\varepsilon_n=\Big(2\cdot\overset{1}{\frac{1}{3}}\cdot\overset{1}{\frac{1}{3}}+0+3\cdot\overset{1}{\frac{1}{3}}\cdot\overset{?3\,(2)}{\frac{2}{3}}+0+1\cdot\overset{(2)}{\frac{2}{3}}\cdot\overset{(2)}{\frac{2}{3}}+$$

$$+2\cdot\frac{2}{3}\cdot\frac{2}{3}+3\cdot\frac{2}{3}\cdot\frac{1}{3}+2\cdot\frac{2}{3}\cdot\frac{2}{3}\Big)10^{-6}=$$

2. a)

$$D=\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix} 10^{-6}$$

$$\overset{I}{\vec{n}}=\overset{(1)}{\frac{2}{3}}\vec{i}+\overset{(2)}{\frac{2}{3}}\vec{j}+\overset{(3)}{\frac{1}{3}}\vec{k} \qquad \overset{II}{\vec{m}}=\overset{(1)}{\frac{1}{\sqrt{2}}}\vec{i}-\overset{(2)}{\frac{1}{\sqrt{2}}}\vec{j}$$

$$\varepsilon_{nm}=\Big(1\cdot\frac{2}{3}\cdot\frac{1}{\sqrt{2}}+2\cdot\frac{2}{3}\Big(-\frac{1}{\sqrt{2}}\Big)+2\cdot\frac{2}{3}\cdot\frac{1}{\sqrt{2}}-1\cdot\frac{2}{3}\cdot\Big(-\frac{1}{\sqrt{2}}\Big)$$

$$+3\cdot\frac{2}{3}\cdot 0+0+3\cdot\frac{1}{3}\cdot\Big(-\frac{1}{\sqrt{2}}\Big)+1\cdot\frac{1}{3}\cdot 0\Big)\cdot 10^{-6}$$

3. $\det D = 0$ под $\det D \neq 0$ линијско

$\det D = 0$ под $\det \neq 0$ равно

$\det D \neq 0$ просторно

b) $D = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 3 \end{bmatrix} \cdot 10^{-6}$

$\det D = 3(12\cdot 3 - 18\cdot 18) - 6(6\cdot 3 - 9\cdot 18) + 9\cdot(6\cdot 18 - 9\cdot 12)$
$\cdot 10^{-6} = 0$

Равно стање деформације

$\varepsilon^3 - I_1 \varepsilon^2 + I_2 \cdot \varepsilon - I_3 = 0$

$I_1 = 18\cdot 10^{-6}$

$I_2 = \begin{vmatrix} 12 & 18 \\ 18 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 9 \\ 9 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ 6 & 12 \end{vmatrix} = -360\cdot 10^{-6}$

$I_3 = \det D = 0$

$\varepsilon^3 - 18\cdot 10^{-6} \varepsilon^2 + (-360)\cdot 10^{-6} \cdot \varepsilon = 0$

(4) $$R(X^\circ; Y^\circ; Z^\circ, t) = \begin{Bmatrix} X^\circ + (X^\circ)^2 t \\ Y^\circ + 2(Y^\circ Z^\circ)^2 t \\ Z^\circ + 3t \end{Bmatrix}$$

a) $$\vec{r} = \begin{Bmatrix} X^{\circ 2} t \\ 2Y^{\circ 2} Z^{\circ 2} t \\ 3t \end{Bmatrix}$$

$$u = \begin{bmatrix} 2X^\circ & 0 & 0 \\ 0 & 4Y^\circ Z^{\circ 2} & 4Y^{\circ 2} Z^\circ \\ 0 & 0 & 0 \end{bmatrix} t$$

$$D = \begin{bmatrix} 2X^\circ & 0 & 0 \\ 0 & 4Y^\circ Z^{\circ 2} & 2Y^{\circ 2} Z^\circ \\ 0 & 2Y^{\circ 2} Z^\circ & 0 \end{bmatrix} t$$

$R = (1; 1; 3)$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 36 & 6 \\ 0 & 6 & 0 \end{bmatrix} t$$

6.

$$\frac{1}{2}\gamma_{xy}$$

$$D = \begin{bmatrix} AXY & XYZ & DYZ \\ XYZ & -2C|Y|^3 & XYZ \\ DYZ & XYZ & AXY \end{bmatrix} 10^{-6}$$

Сент - Венан - ови услови равнотеже

$$(1) \quad \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z}$$

$$\frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{zx}}{\partial z \partial x}$$

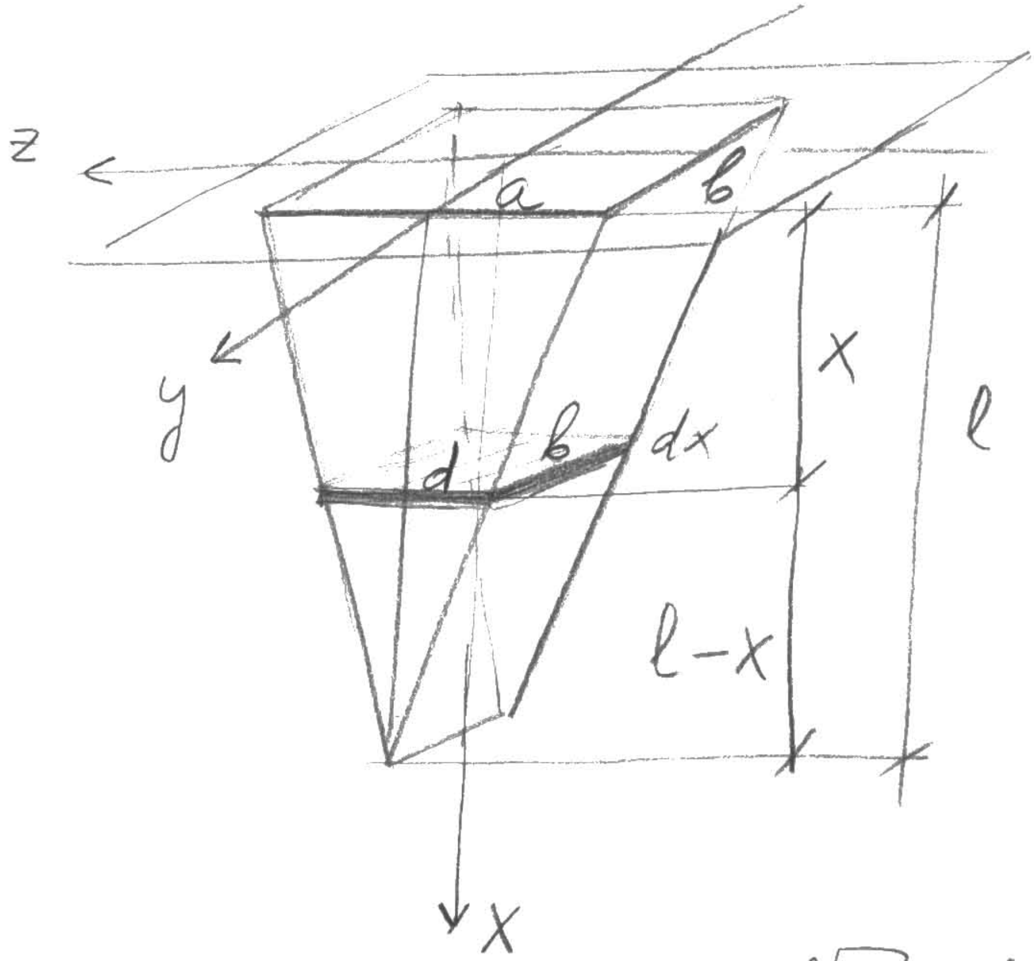
$$\frac{\partial^2 \varepsilon_x}{\partial y \partial z} = \frac{1}{2}\frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yx}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_y}{\partial z \partial x} = \frac{1}{2}\frac{\partial}{\partial y}\left(-\frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yx}}{\partial x}\right)$$

$$\frac{\partial^2 \varepsilon_z}{\partial x \partial y} = \frac{1}{2}\frac{\partial}{\partial z}\left(-\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yx}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y}\right)$$

$$(1) \quad 0 + 0 = 2z$$

Аксијално напрезање



$$d : (l-x) = a : l \qquad \underline{d \cdot l = a(l-x)}$$

a) $$d = \frac{a(l-x)}{l}$$

$$A = d \cdot b = \frac{ab(l-x)}{l}$$

$$q_x = \gamma \cdot A = \gamma d b = \gamma \frac{ab(l-x)}{l}$$

$$Q_x = q_x \cdot dx = \gamma \frac{ab(l-x)}{l} dx$$

b) $$N(x=l) = 0$$

$$\underline{u(x=0) = 0}$$

c) $N(x) = \int_x^l q(x) \cdot dx = \int_x^l \gamma \frac{ab(l-x)}{l} dx + C_1$

$$\sigma_x = \frac{N(x)}{A} = \frac{N(x)}{\frac{ab(l-x)}{l}}$$

$$u(x) = \int \varepsilon_x \, dx = \int \frac{\sigma_x}{E} dx$$

Конститутивне везе

1. a)

$$\sigma_x = 2 \cdot 84000 \cdot 2 \cdot 10^{-6} + 84000 \cdot 4 \cdot 10^{-6} = 0{,}672 \text{ MPa}$$

$$\sigma_y = 2 \cdot 84000 \cdot 2 \cdot 10^{-6} + 84000 \cdot 4 \cdot 10^{-6} = 0{,}672 \text{ MPa}$$

$$\sigma_z = 2 \cdot 84000 \cdot 0 + 84000 \cdot 4 \cdot 10^{-6} = 0{,}336 \text{ MPa}$$

$$\tau_{xy} = \mu \cdot \gamma_{xy} = 84000 \cdot 0 = 0$$

$$\tau_{xz} = \mu \cdot \gamma_{xz} = 84000 \cdot 2 \cdot 10^{-6} = 0{,}168 \text{ MPa}$$

$$\tau_{yz} = \mu \cdot \gamma_{yz} = 84000 \cdot 2 \cdot 10^{-6} = 0{,}168 \text{ MPa}$$

$$S = \begin{array}{c} \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \end{array} \\ \begin{bmatrix} 0{,}672 & 0 & 0{,}168 \\ 0 & 0{,}672 & 0{,}168 \\ 0{,}168 & 0{,}168 & 0{,}336 \end{bmatrix} \end{array} \text{MPa} \begin{array}{l} \longrightarrow p_x \\ \longrightarrow p_y \\ \longrightarrow p_z \end{array}$$

$$\vec{n} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$

$$p_x = \frac{1}{\sqrt{2}} \cdot 0{,}672 = 0{,}475$$

$$p_y = \frac{1}{\sqrt{2}} \cdot 0{,}672 = 0{,}475$$

$$p_z = \frac{1}{\sqrt{2}} \cdot 0{,}168 + \frac{1}{\sqrt{2}} \cdot 0{,}168 = 0{,}237 \text{ MPa}$$

$$\vec{p} = \{0{,}475;\ 0{,}475;\ 0{,}237\} \text{ MPa}$$

2.

$$S = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix} MPa$$

$$\vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{3}{5\sqrt{3}}\vec{j} + \frac{4}{5\sqrt{3}}\vec{k} \qquad E = 70\,GPa$$

$$\nu = 0{,}15$$

$$G = \frac{E}{2(1+\nu)} = \frac{70\,000\,MPa}{2(1+0{,}15)} = 30\,434{,}78\,MPa$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)] \qquad \gamma_{xz} = \frac{1}{G}\cancel{\gamma_{xz}}\,\tau_{xz}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \qquad \gamma_{yz} = \frac{1}{G}\cancel{\gamma_{yz}}\,\tau_{yz}$$

$$\varepsilon_x = \frac{1}{70000}[3 - 0{,}15(1-1)] = 4{,}286 \cdot 10^{-5}$$

$$\varepsilon_y = \frac{1}{70000}[1 - 0{,}15(3-1)] = 1 \cdot 10^{-5}$$

$$\varepsilon_z = \frac{1}{70000}[-1 - 0{,}15(3+1)] = -2{,}285 \cdot 10^{-5}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{1}{30434{,}78} \cdot 0 = 0$$

$$\gamma_{xz} = \frac{1}{G} \cdot \tau_{xz} = \frac{1}{30434{,}78} \cdot 2 = \quad \rightarrow \frac{1}{2}\gamma_{xz}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz} = \frac{1}{30434{,}78}(-1) = \quad \frac{1}{2}\gamma_{yz}$$

D

↓

ε_n

3 9

$$S = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} MPa$$

$\varepsilon_x = 3 \cdot 10^{-6}$ $\varepsilon_y = -1 \cdot 10^{-6}$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)]$$

$$\left.\begin{aligned} 3 \cdot 10^{-6} &= \frac{1}{E}[4 - \nu(1+1)] \\ -1 \cdot 10^{-6} &= \frac{1}{E}[1 - \nu(1+4)] \end{aligned}\right\}$$

$$E = \frac{10^6}{3}[4 - 2\nu]$$

$$E = -10^6[1 - 5\nu]$$

4. $\varepsilon_{n1} = 20 \cdot 10^{-6}$ $\varepsilon_{n2} = 5 \cdot 10^{-6}$ $\varepsilon_{n3} = 15 \cdot 10^{-6}$

$\varphi_1 = 20°$ $\varphi_2 = 65$ $\varphi_3 = 110$

$$\left.\begin{aligned} \varepsilon_{n1} &= \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 40 + \frac{1}{2} \gamma_{xy} \sin 40 \\ \varepsilon_{n2} &= \text{-----} \cos 130 + \text{-----} \sin 130 \\ \varepsilon_{n3} &= \cos 220 + \text{---} \sin 220 \end{aligned}\right\} \begin{matrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{matrix}$$

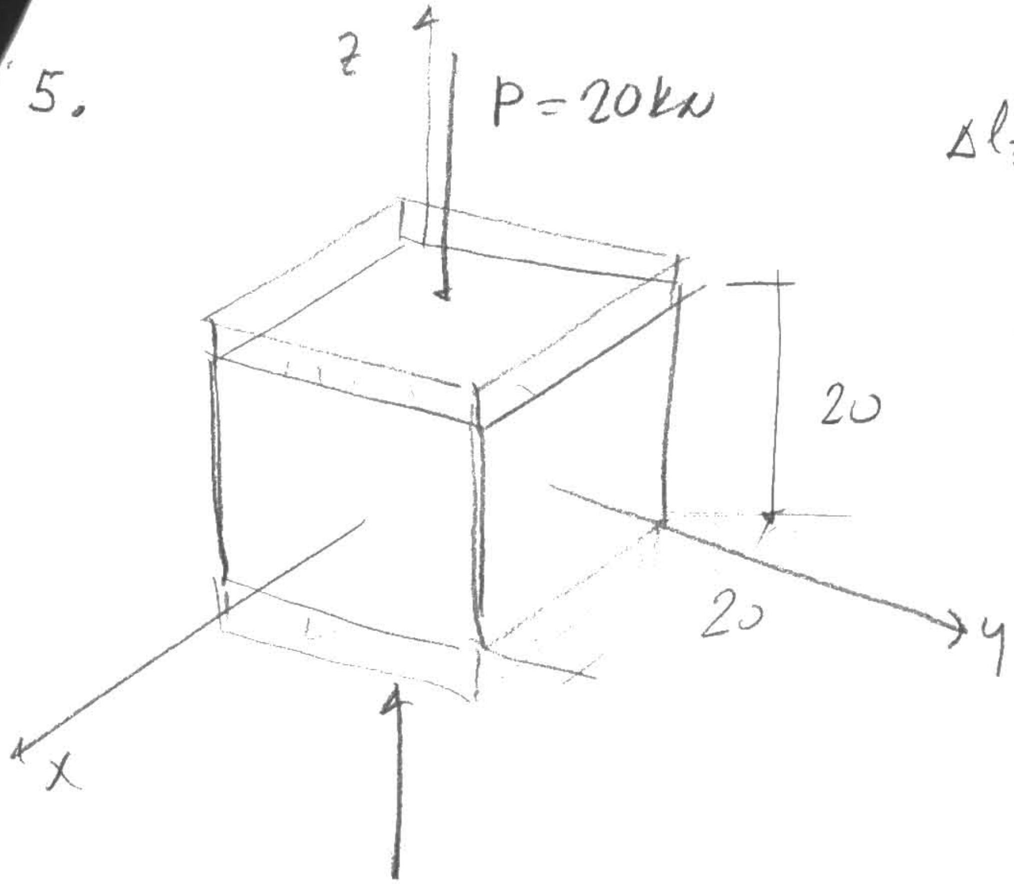
a)
$$D = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b)
$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{1}{2}\gamma_{xy}\right)^2}$$

c) $e = \varepsilon_x + \varepsilon_y$

d) E u V

5.



$\Delta l_z = -0{,}3\,cm$

$\Delta l_x = \Delta l_y = 0{,}1\,cm$

$\sigma_z = -\frac{P}{A} = -\frac{20\,kN}{400\,cm^2} = -0{,}05\,\frac{kN}{cm^2}$

$\sigma_x = \sigma_y = 0$

$\varepsilon_z = \frac{\Delta l_z}{l_z} = -\frac{0{,}3\,cm}{20\,cm} = -0{,}015$

$\varepsilon_x = \varepsilon_y = \frac{0{,}1\,cm}{20\,cm} = 0{,}005$

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\,\cancel{(\sigma_x + \sigma_y)}^{\,0}\right]$

$-0{,}015 = \frac{1}{E}\left(-0{,}05\,\frac{kN}{cm^2}\right)$

модул еласт.

$E = 3{,}3333\,\frac{kN}{cm^2}$

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

$0{,}005 = \frac{1}{3{,}333}\left[0 - \nu(0 - 0{,}05)\right]$

Поисонов коеф

$\nu = \frac{1}{3}$

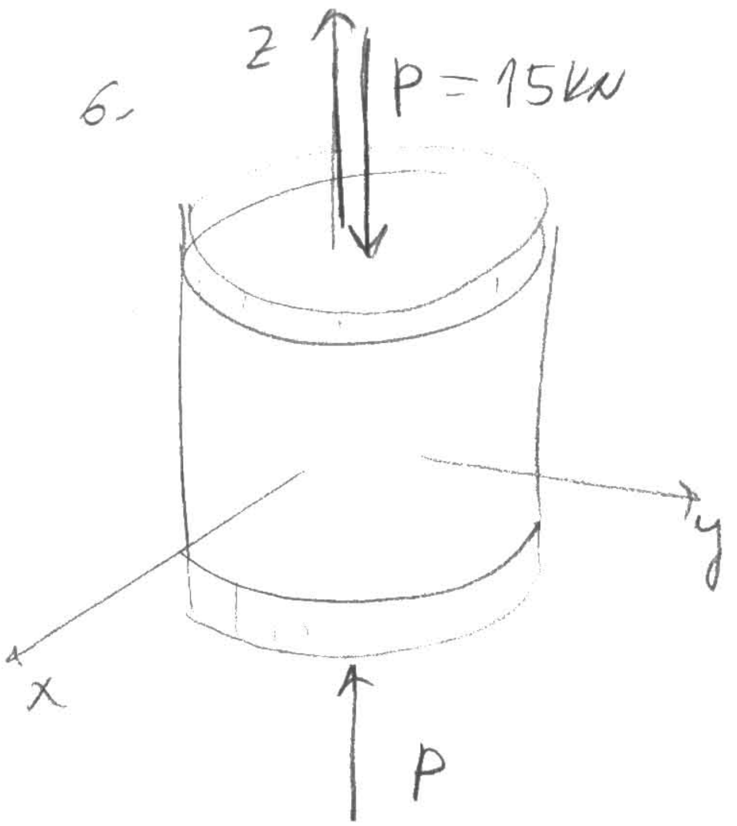
модул смицање

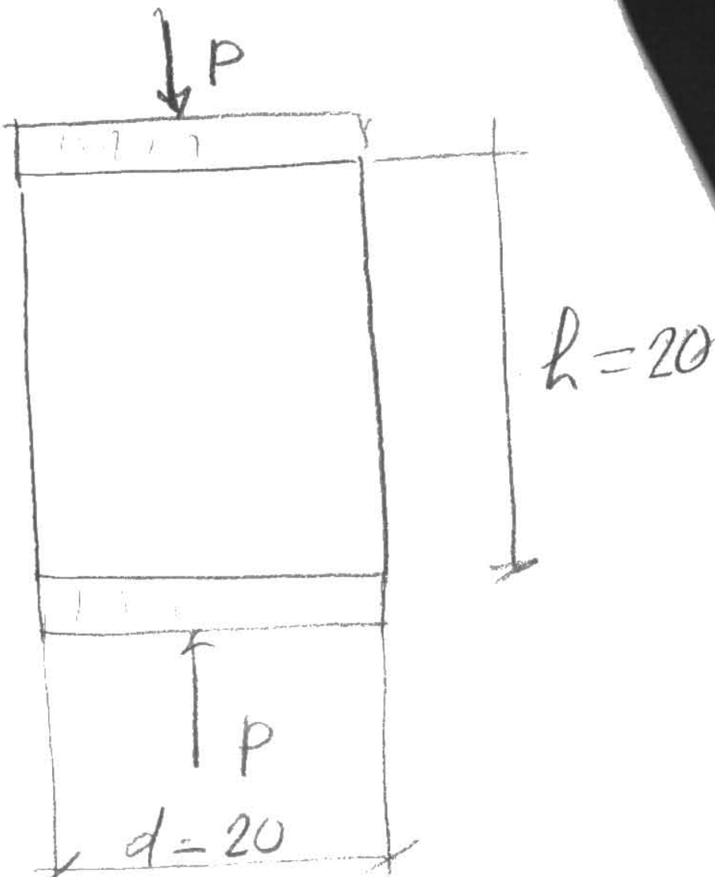
$G = \frac{E}{2(1+\nu)}$

модул ком

$K = \frac{E}{3(1-2\nu)}$

6.





$\sigma_x = \sigma_y = 0$

$\Delta l_z = \underline{\underline{-0{,}02\ cm}}$

$\sigma_z = -\frac{15\ kN}{100\pi\ cm^2} = -0{,}0477\ \frac{kN}{cm^2}$

$\Delta l_x = \Delta l_y = \underline{\underline{0{,}007\ cm}}$

$\varepsilon_z = \frac{-0{,}02}{20\ cm} = -0{,}001$

$\varepsilon_x = \varepsilon_y = \frac{0{,}007}{20} = 0{,}00035$

$\varepsilon_z = \frac{1}{E}[-0{,}0477 - \cancel{\nu(\sigma_x + \sigma_y)}^{0}] \Rightarrow E$

$\varepsilon_x = \frac{1}{E}[0 - \nu(0 - 0{,}0477)] \Rightarrow \nu$

b) $\sigma_z = -0{,}0477 \quad E, \nu \quad \varepsilon_x = \varepsilon_y = 0 \quad \underline{\sigma_x = \sigma_y}$

$\varepsilon_z = \frac{\Delta l_z}{l_z} \quad \underline{\Delta l_z = \varepsilon_z l_z} = \varepsilon_z \cdot 20\ cm$

$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \varepsilon_z = \frac{1}{E}[\sigma_z + \nu(\sigma_x + \sigma_y)]$

$0 = \frac{1}{E}[\sigma_x - \nu(\sigma_x - 0{,}0477)] \Rightarrow \sigma_x = \sigma_y$

7. a) $\varepsilon_y = 0 \qquad \sigma_x = 0 \qquad \sigma_z = -5 \frac{kN}{m^2} \qquad E, \nu$ (12)

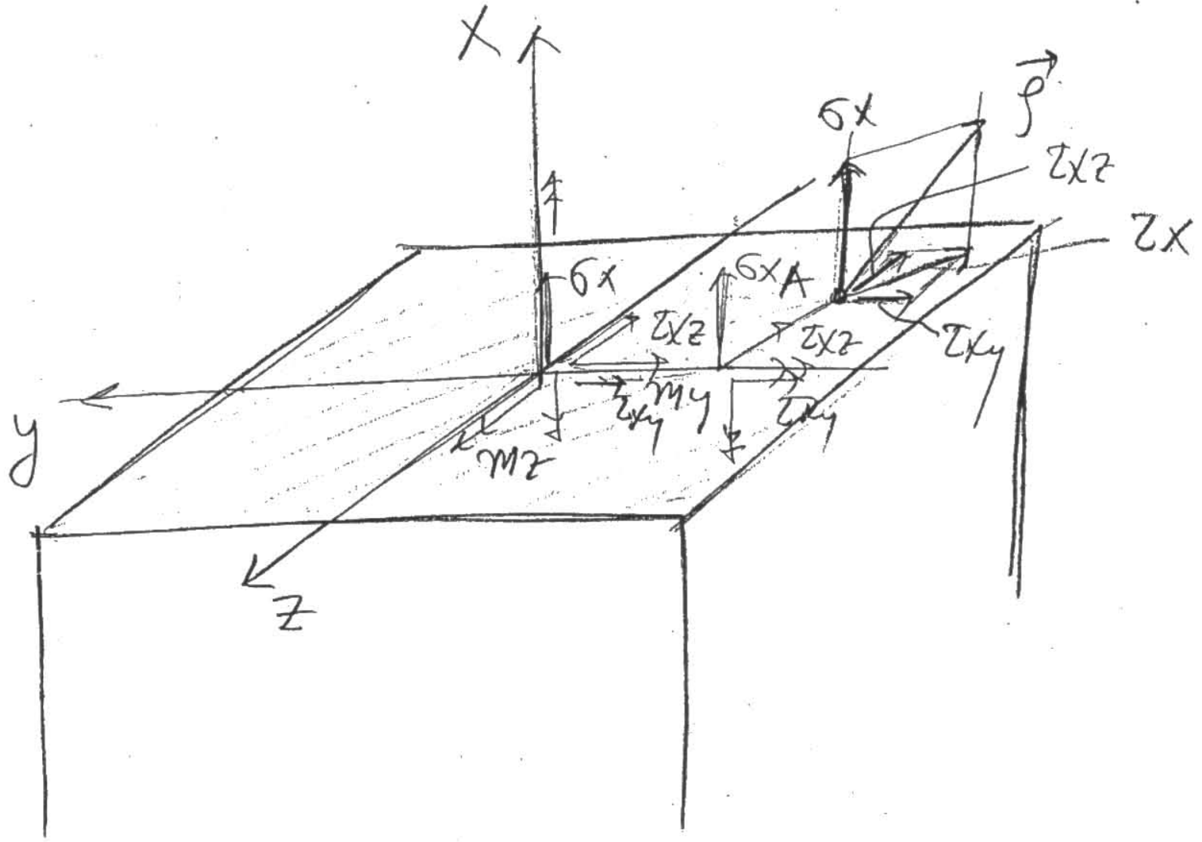
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \Rightarrow \sigma_y$$

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & -5 \end{bmatrix} \frac{kN}{m^2} \Rightarrow D$$

b) $\varepsilon_y = 0 \qquad \sigma_x = -2{,}5 \frac{kN}{m^2} \qquad \sigma_z = -5 \quad E, \nu$

Нормални напони

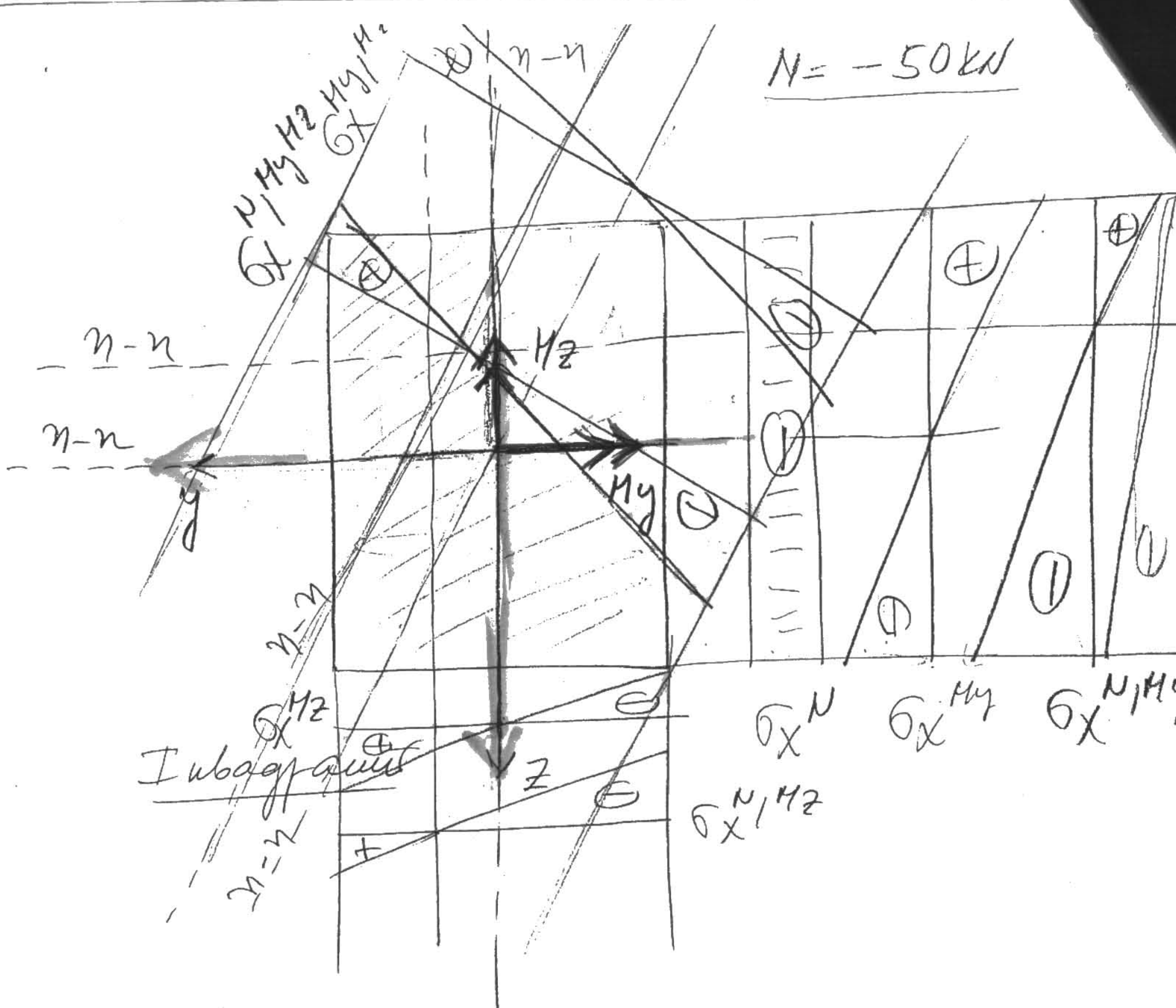
напон σ_x



$N, M_y, M_z \longrightarrow \sigma_x$

$T_y, T_z, M_t \longrightarrow \tau_x$

Н. Јовановић



1) $\sigma_x^N = \frac{N}{A} = \frac{-50\,kN}{A} = -a$

десно што притиска I к...

2) $\sigma_x^{My} = \frac{My}{I_y} z = - \frac{My}{I_y} z = -b \cdot z$ — КОНСТАНТА

n-n линија

$\sigma_x = 0 \quad -b \cdot z = 0 \quad z = 0$

3) $\sigma_x^{Mz} = \frac{Mz}{I_z} y = c \cdot y$

n-n оса

$\sigma_x = 0 \quad c \cdot y = 0 \quad y = 0$

$$\sigma_x^{N,My} = \frac{N}{A} - \frac{M_y}{I_y} z = -a - b \cdot z$$

<u>n-n оса</u>

$\sigma_x = 0 \quad -a - bz = 0 \quad \underline{z = -a/b}$

5) $$\sigma_x^{N,Mz} = \frac{N}{A} + \frac{M_z}{I_z} \cdot y = -a + c \cdot y$$

<u>n-n оса</u>

$\sigma_x = 0 \quad -a + cy = 0 \quad y = a/c$

6) $$\sigma_x^{My,Mz} = -\frac{M_y}{I_y} z + \frac{M_z}{I_z} \cdot y = -bz + c \cdot y$$

<u>n-n оса</u>

$\sigma_x = 0 \quad -bz + cy = 0$

y	0	3
z	0	7

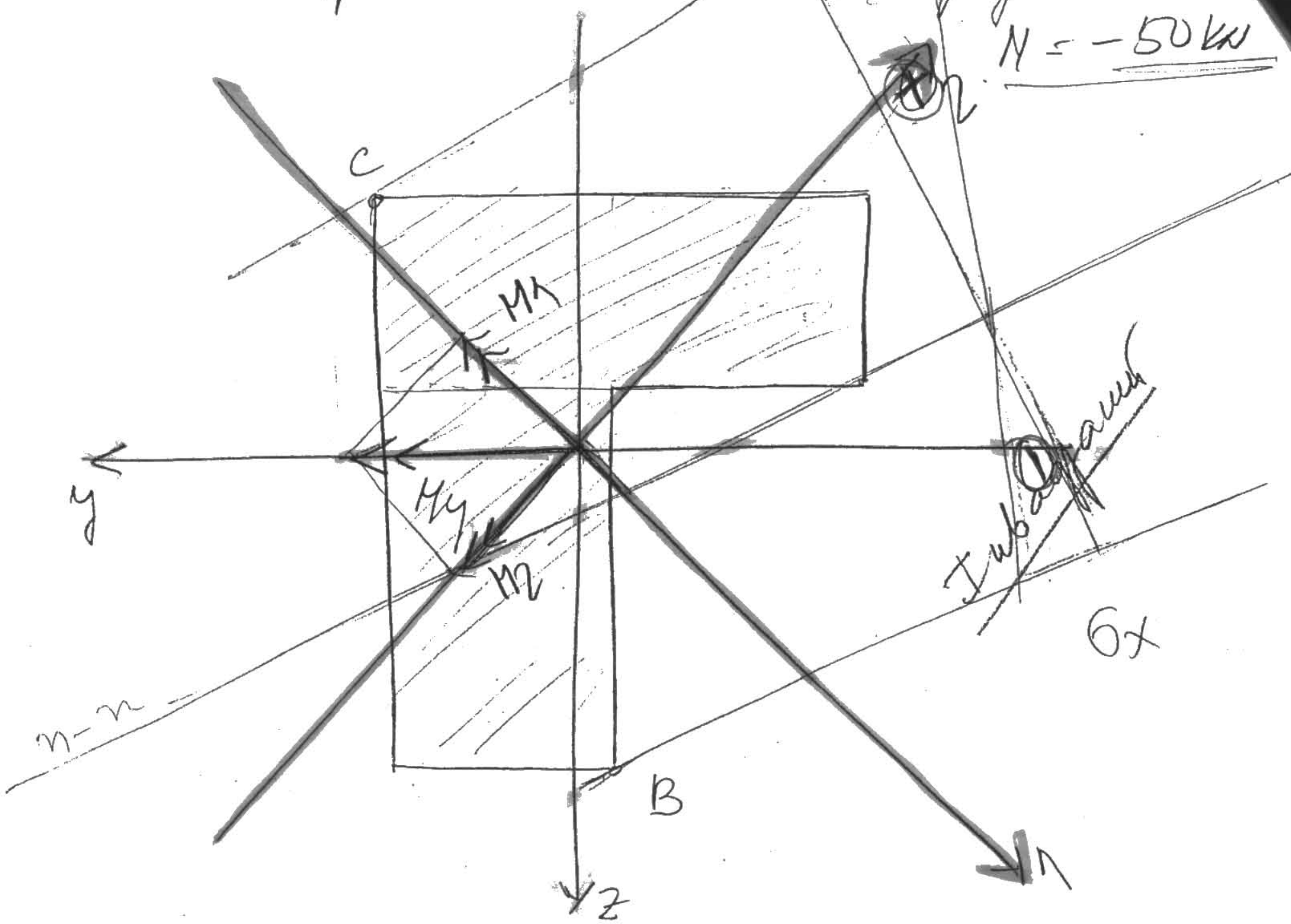
7) $$\sigma_x^{N,My,Mz} = \frac{N}{A} - \frac{M_y}{I_y} z + \frac{M_z}{I_z} \cdot y = -a - bz + cy$$

<u>n-n оса</u>

$\sigma_x = 0 \quad -a - bz + cy = 0$

y	a/c	0
z	0	$-a/b$

Пресек без осе симетрије



$$\sigma_x = \frac{N}{A} - \frac{M_1}{I_1}(2) + \frac{M_2}{I_2}(1) = -a - b(2) + c(1)$$

n−n оса $\sigma_x = 0$

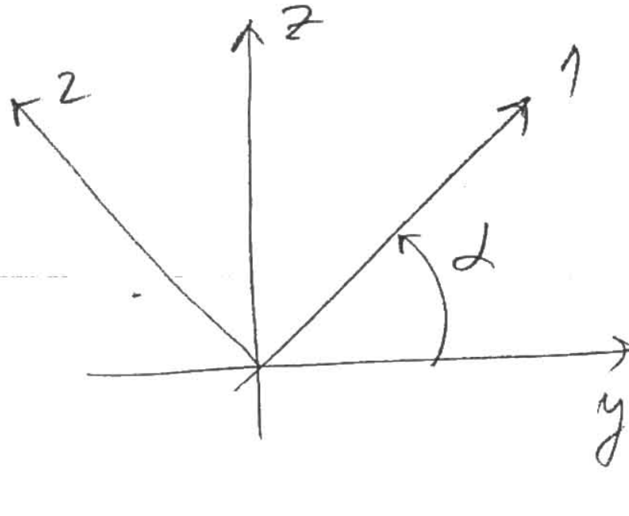
$$-a - b(2) + c(1) = 0$$

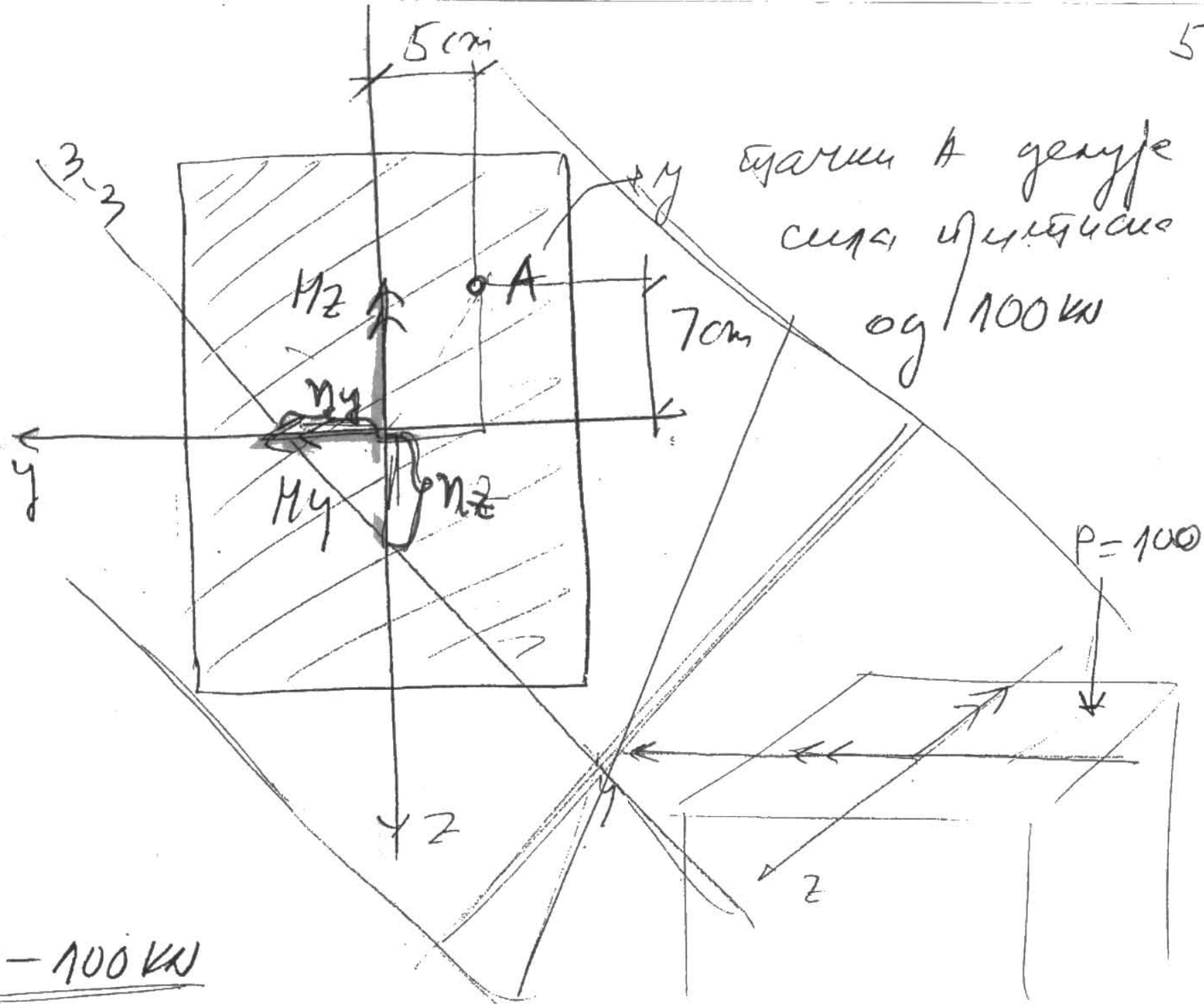
(1)	0	a/c
(2)	−a/b	0

$B(y_B; z_B)$ $C(y_C; z_C)$

$$1 = y\cos\alpha + z\sin\alpha$$

$$2 = z\cos\alpha - y\sin\alpha$$





у тачки А делује
сила притиска
од 100 kN

$\underline{N = -100\ kN}$

$M_y = 100\ kN \cdot 7\ cm = 700\ kNcm$

$M_z = 100\ kN \cdot 5\ cm = 500\ kNcm$

$\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} \cdot y = -a + bz + cy$

n-n оса $\sigma_x = 0$ $-a + bz + cy = 0$

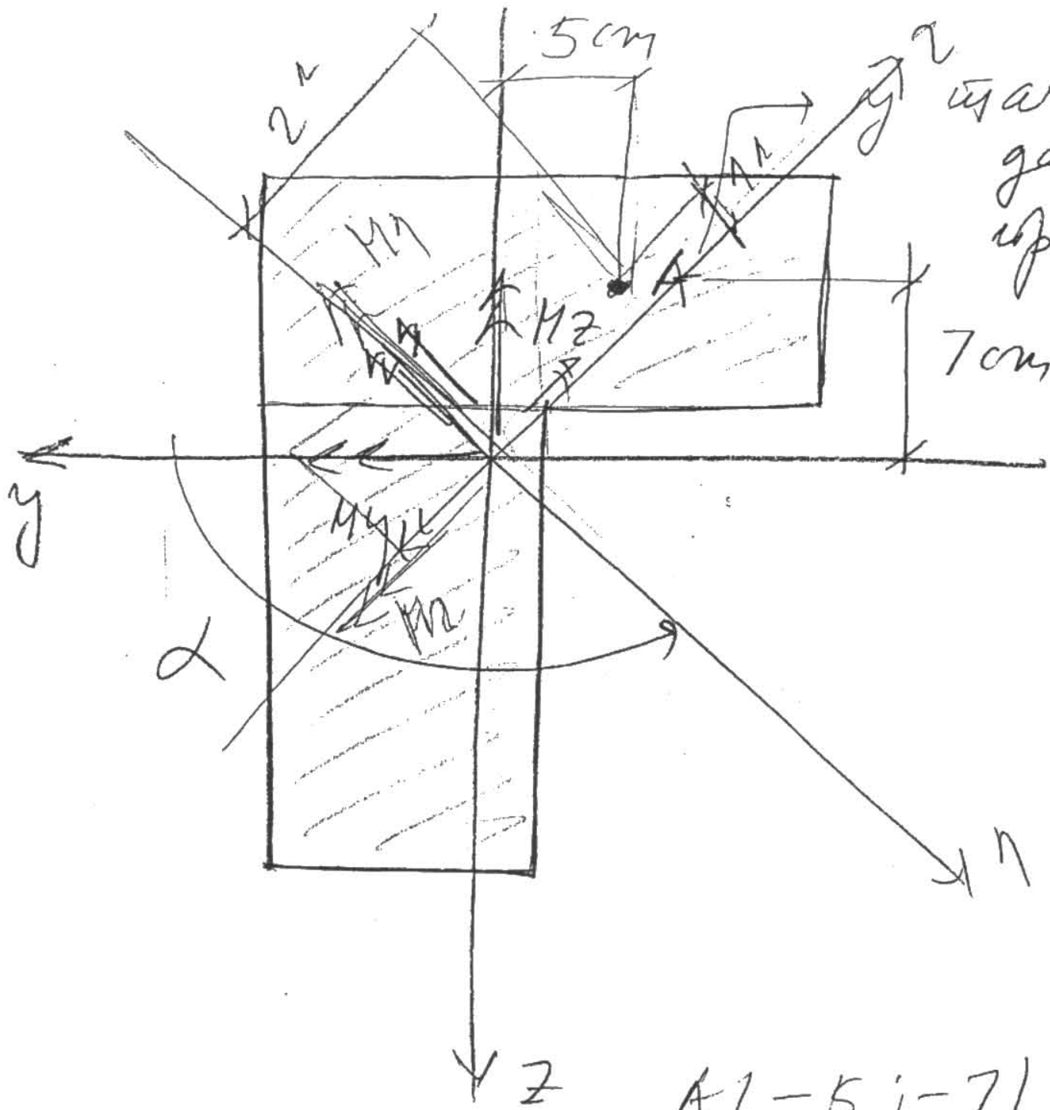
y	0	a/c
z	a/b	0

n-n оса $A(-5; -7)$

$n_y = -\frac{i_z^2}{y_A}$

$n_z = -\frac{i_y^2}{z_A}$

тачки A делује сила притиска од 100kN



$A(-5; -7)$

$1^A = y\cos\alpha + z\sin\alpha$

$2^A = z\cos\alpha - y\sin\alpha$

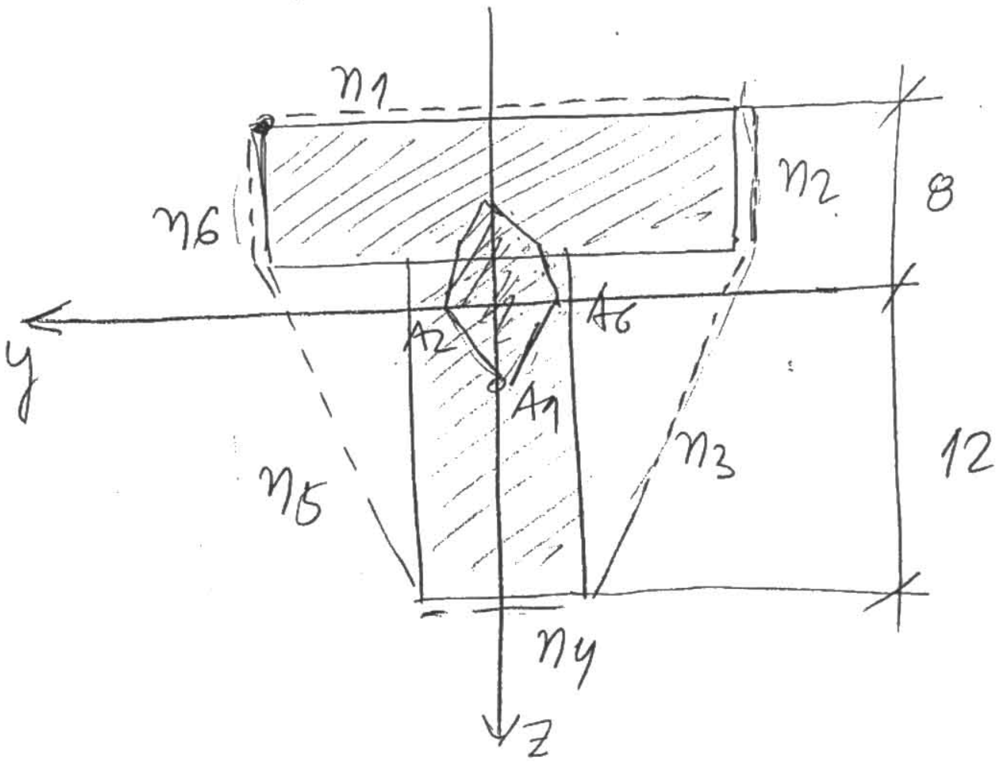
$N = -100\,kN$

$M_y = 700\,kNcm$

$M_z = 500\,kNcm$

$$\sigma_x = \frac{N}{A} - \frac{M_1}{I_1}(2) + \frac{M_2}{I_2}(1)$$

Језгро пресека



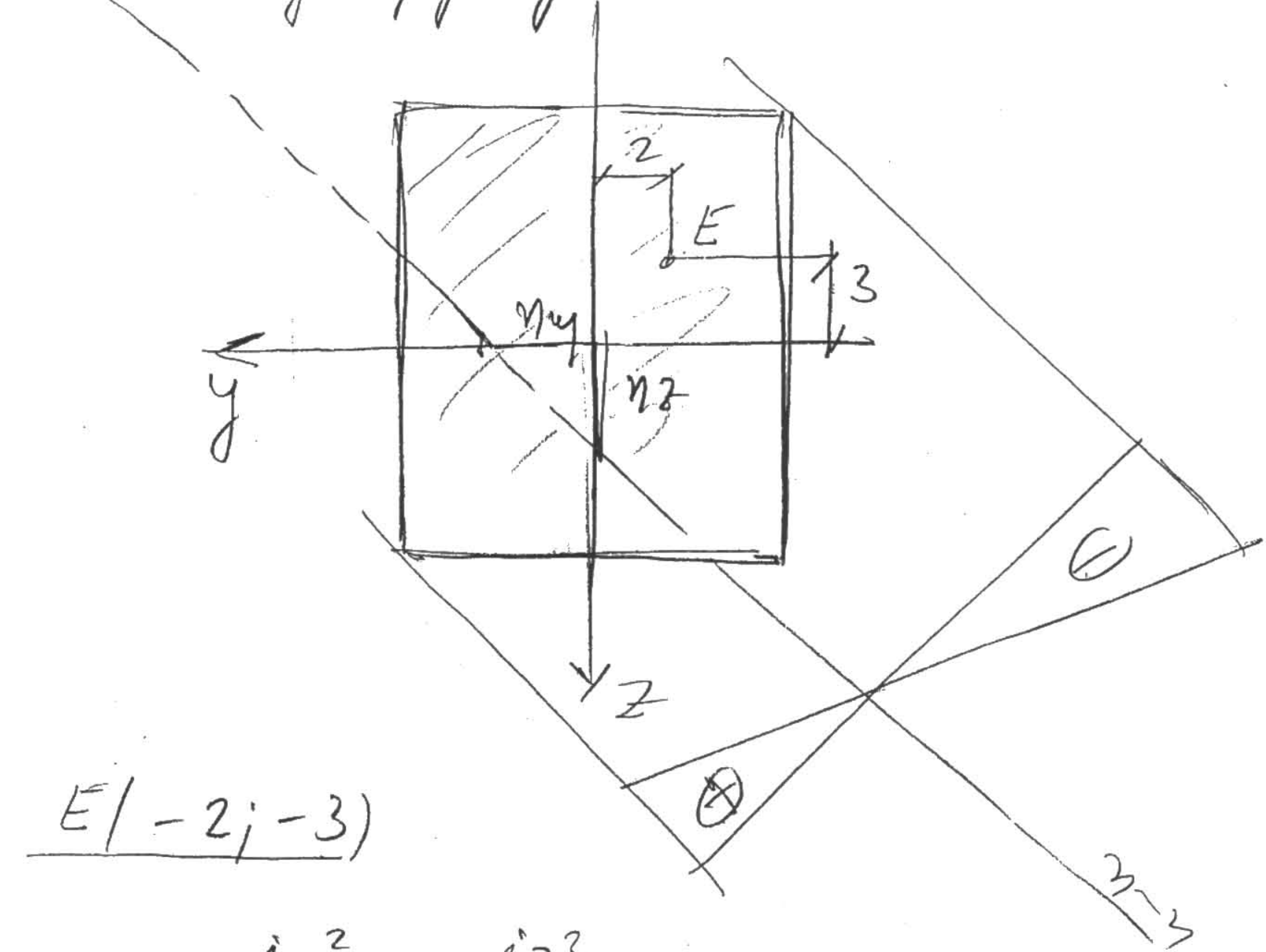
$n_1 - n_1 \qquad A_1(y_1; z_1) \qquad A_1(0; 1{,}2)$

$n_y = \infty, \quad n_z = -8\ cm$

$y_1 = -\frac{i_z^2}{n_y} = 0$

$z_1 = -\frac{i_y^2}{n_z} = -\frac{i_y^2}{-8} = 1{,}20$

Да ли тачка $E(y_E, z_E)$ припада језгру пресека?



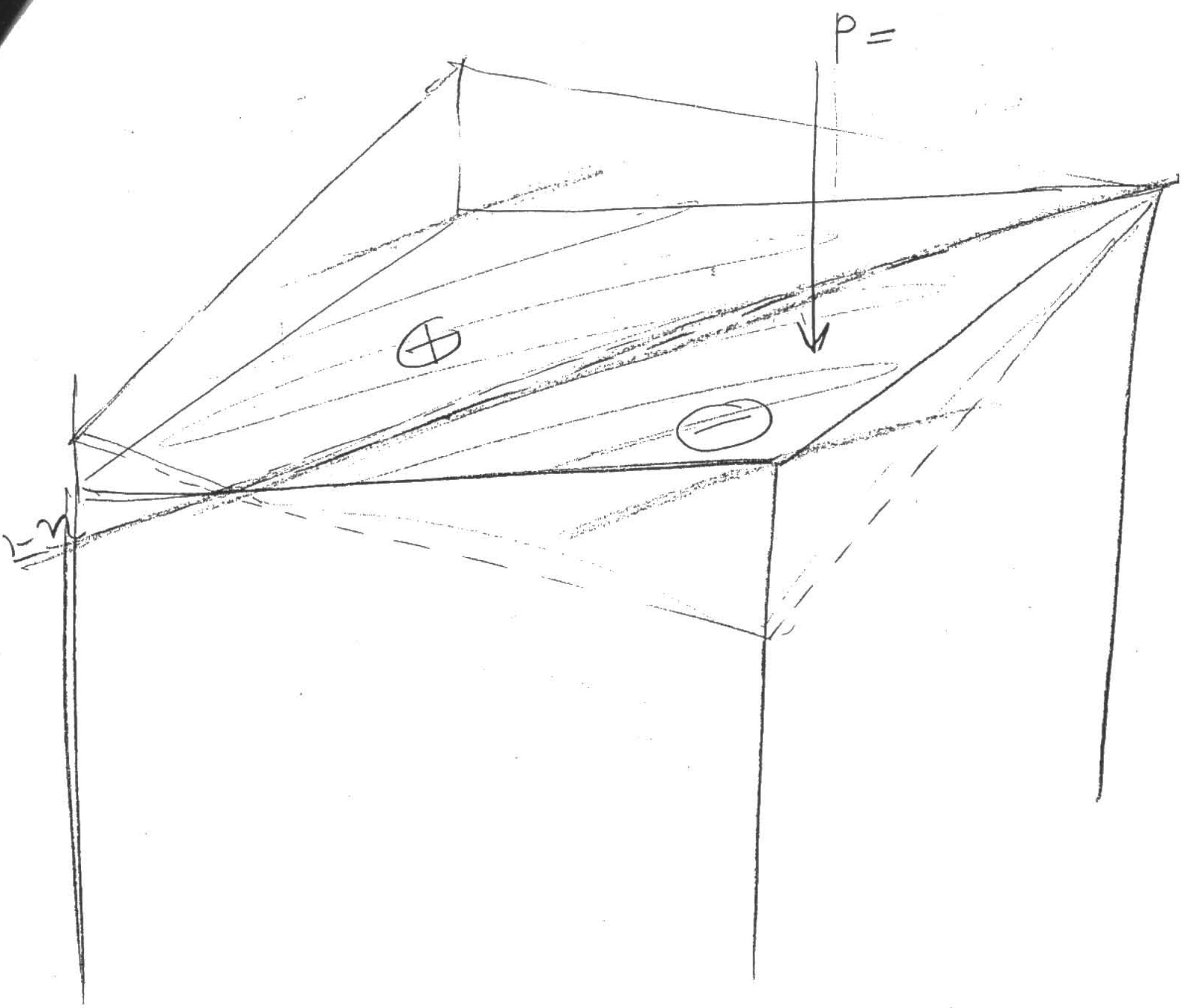
$E(-2; -3)$

$$n_y = -\frac{i_z^2}{y_E} = -\frac{i_z^2}{-2} = 3,5$$

$$n_z = -\frac{i_y^2}{z_E} \ldots$$

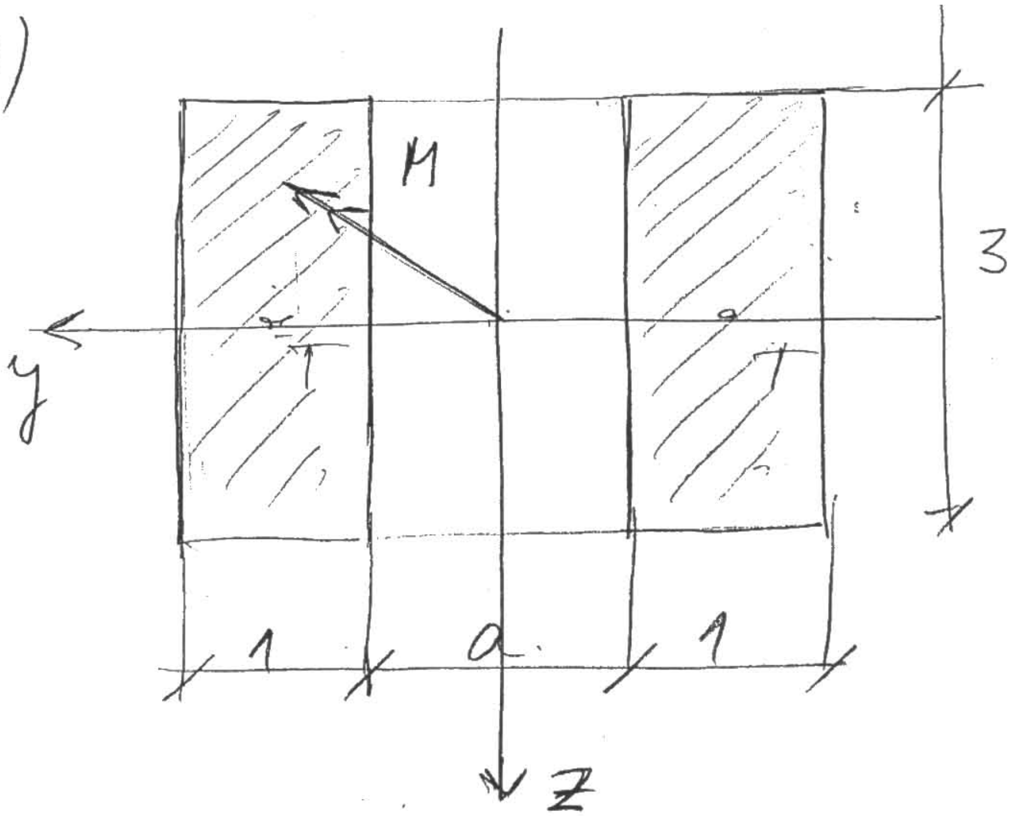
Пошто цела n–n оса сече пресек тачка E не припада језгру!

n – n осе



Чисто право савијање – чисто косо савијање

1)



$\underline{\underline{I_y = I_z}}$

$$I_y = 2 \cdot \frac{1 \cdot 3^3}{12} = \frac{27}{6} = \frac{9}{2} = 4{,}5\ cm^4$$

$$I_z = 2 \cdot \left[\frac{3 \cdot 1^3}{12} + 3 \cdot 1 \cdot \left(\frac{1}{2} + \frac{a}{2} \right)^2 \right]$$

$$\cancel{2}\left[\frac{1}{4} + 3 \cdot \left(0{,}5 + \frac{a}{2} \right)^2 \right] = \cancel{4{,}5}^{\,2{,}25} \quad /:2$$

$$a =$$

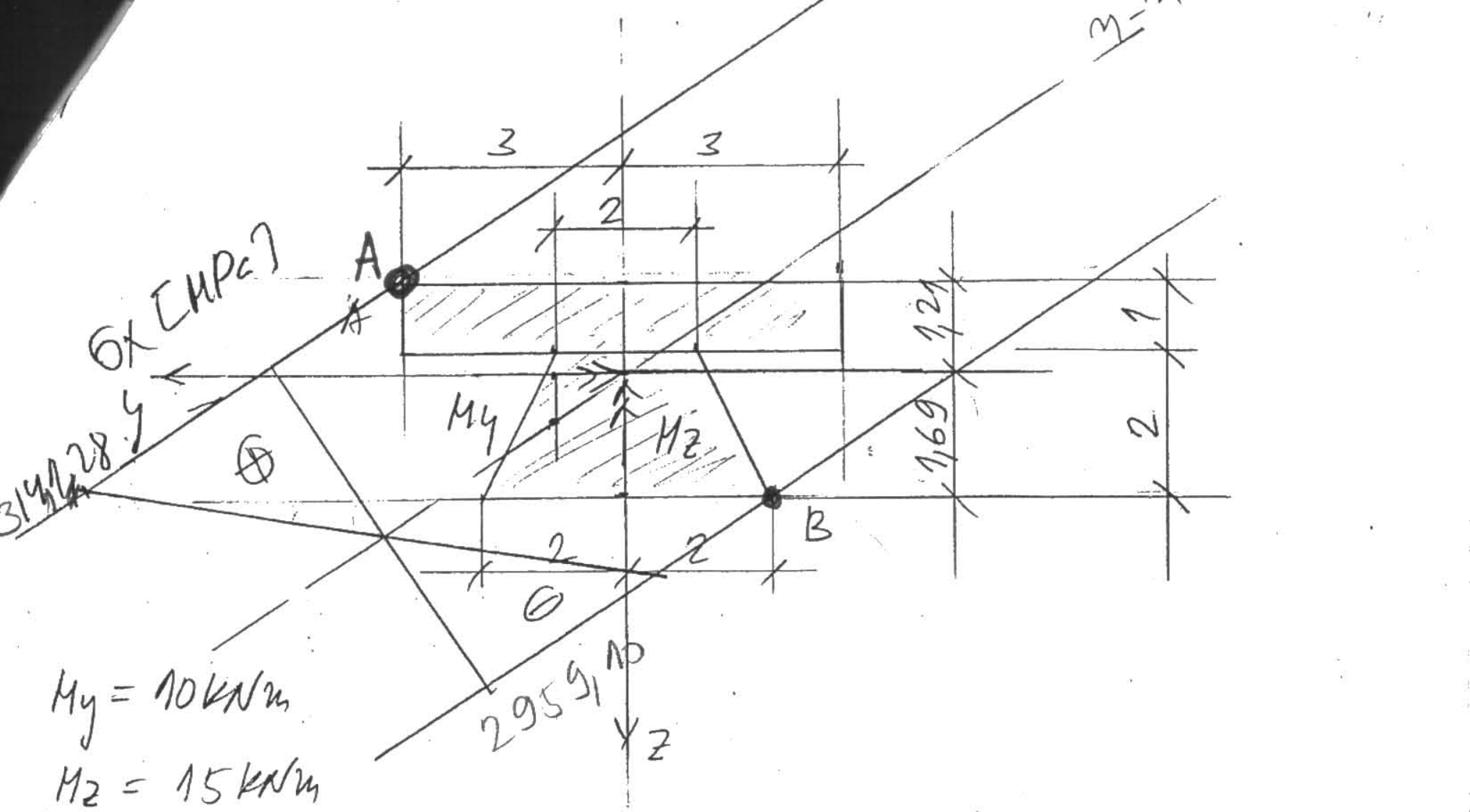
$$\frac{1}{4} + 3 \cdot \left(0{,}25 + 0{,}5a + \frac{a^2}{4} \right) = 2{,}25$$

$$\frac{1}{4} + 0{,}75 + 1{,}5a + \frac{3}{4}a^2 = 2{,}25 \; / \cdot 4$$

$$4 + 6a + 3a^2 = 10 \qquad a_{1/2} = \frac{-2 \pm \sqrt{4+8}}{2}$$

$$3a^2 + 6a - 6 = 0 \qquad a_1 = 0{,}73 \quad a_2 = -2{,}73$$

$$a^2 + 2a - 2$$



$M_y = 10\,kNm$

$M_z = 15\,kNm$

$I_y = 10,213\,cm^4$

$I_z = 23,0\,cm^4$

a) $\sigma_x = -\frac{M_y}{I_y} z + \frac{M_z}{I_z} \cdot y = -\frac{1000\,kNcm}{10,213\,cm^4} z + \frac{1500\,kNcm}{23\,cm^4} y$

b) $W_y^{gore} = \frac{I_y}{z_{max}} = \frac{10,213\,cm^4}{1,69\,cm} = 6,04\,cm^3$

$W_y^{dole} = \frac{I_y}{z^{dole}} = \frac{10,213\,cm^4}{1,21\,cm} = 8,44\,cm^3$

$W_z = \frac{I_z}{y_{max}} = \frac{23\,cm^4}{3\,cm} = 7,66\,cm^3$

c) $\sigma_x = 0 \quad -\frac{1000}{10,213} z + \frac{1500}{23} \cdot y = 0$

y	0	1
z	0	966

d) $A(3; -1,21)$

$\sigma_x^A = -\frac{1000 \cdot (-1,21)}{10,213} + \frac{1500}{23} \cdot 3 = 314,128 \frac{kN}{cm^2}$

Единици за напон

$$1\,GPa = 10^3\,MPa = 10^6\,kPa\left[\frac{kN}{m^2}\right] = 10^9\,Pa\left[\frac{N}{m^2}\right]$$

$$\boxed{1\,\frac{kN}{cm^2} = 10\,MPa}$$

1

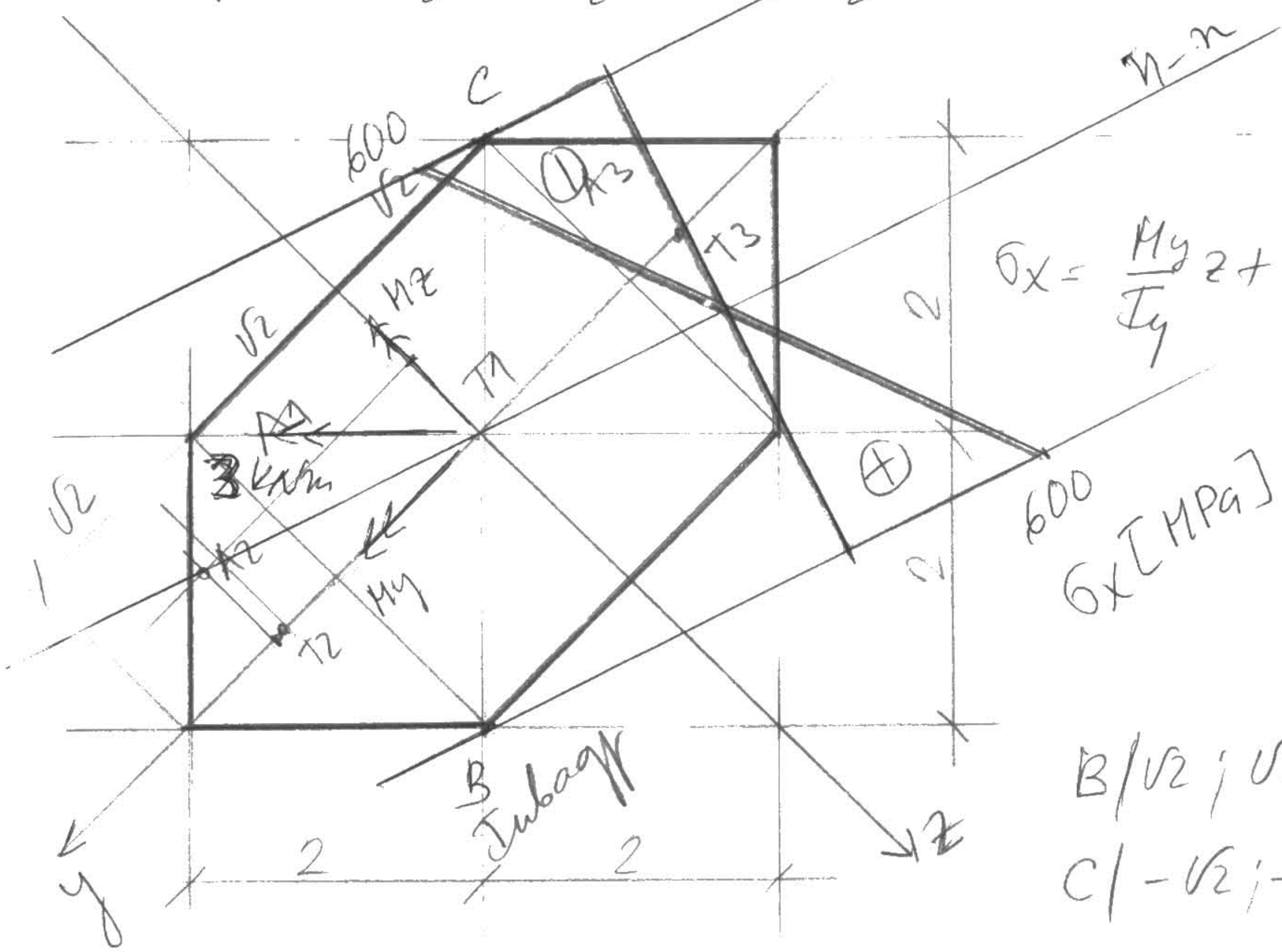
$B(-2; 1{,}69)$

$$\sigma_x^B = -\frac{1000}{10{,}213} \cdot 1{,}69 + \frac{1500}{23} \cdot (-2) \frac{kN}{cm^2} = -295{,}910 \frac{kN}{cm^2}$$

$$\sigma_x^B = -2959{,}10 \ MPa$$

e) $$\tau_{max} = \frac{|\sigma_{max}|}{2} = \frac{3141{,}28 \ MPa}{2} = 1570{,}64 \ MPa$$

$M_y = M_z = \frac{\sqrt{2}}{2} M = \frac{\sqrt{2}}{2} \cdot 3\,kNm = \frac{3}{2}\sqrt{2}\,kNm = 150\sqrt{2}\,kNcm$



$\sigma_x = \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$

$B(\sqrt{2}; \sqrt{2})$

$C(-\sqrt{2}; -\sqrt{2})$

$A_2 = \frac{1}{2} \cdot 2 \cdot 2 = 2 \qquad T_2\left(\frac{4}{3}\sqrt{2}; 0\right)$

$A_3 = 2 \qquad T_3\left(-\frac{4}{3}\sqrt{2}; 0\right)$

$I_y = \frac{(2\sqrt{2})^4}{12} + 2 \cdot \frac{\sqrt{2}\,(2\sqrt{2})^3}{48} = 6{,}667\,cm^4$

$I_z = \frac{(2\sqrt{2})^4}{12} + 2\left[\frac{2\sqrt{2}\,(\sqrt{2})^3}{36} + 2 \cdot \left(\frac{4\sqrt{2}}{3}\right)^2\right] = 20\,cm^4$

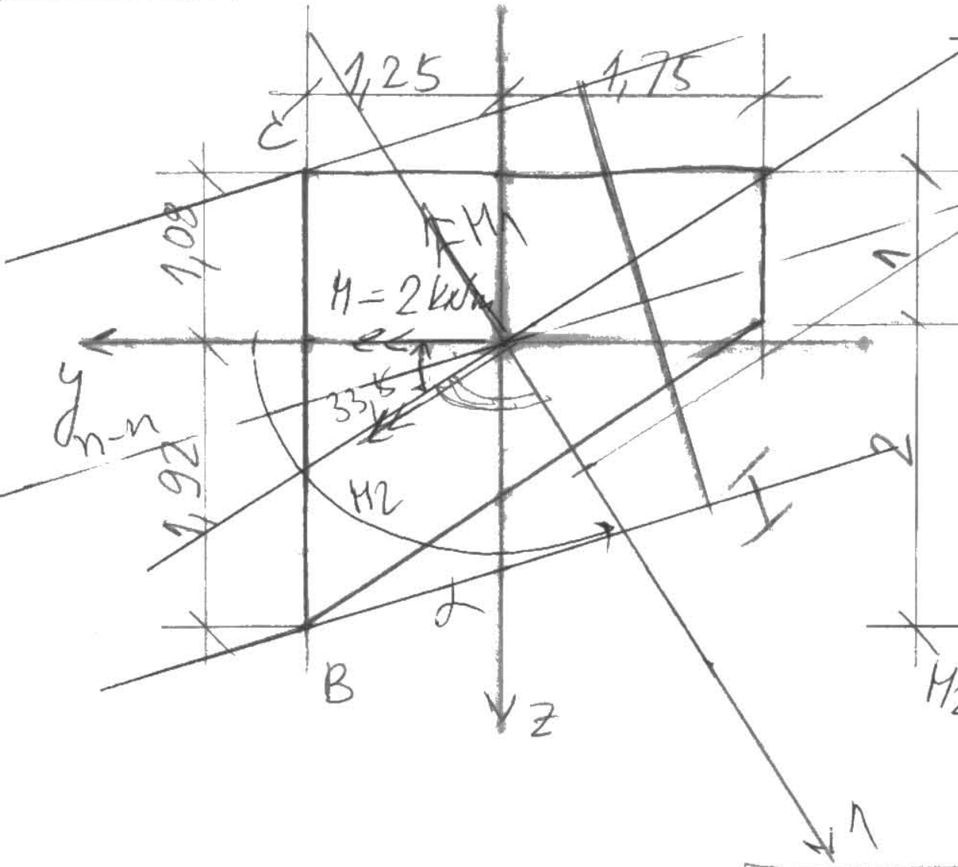
y	0	2
z	0	-0,667

$\sigma_x = \frac{150\sqrt{2}\,kNcm}{6{,}667\,cm^4} z + \frac{150\sqrt{2}\,kNcm}{20\,cm^4} y$

n-n osa $\sigma_x = 0 \qquad \frac{150\sqrt{2}}{6{,}667} z + \frac{150\sqrt{2}}{20} y = 0$

$\sigma_x^B = \frac{150\sqrt{2}}{6{,}667} \cdot \sqrt{2} + \frac{150\sqrt{2}}{20} \cdot \sqrt{2}\ \frac{kN}{cm^2} = 60\,\frac{kN}{cm^2} = 600\,MPa$

$\sigma_x^C = -600\,MPa$



$I_y = 2,9583$

$I_z = 4,1250$

$I_{yz} = 1,3750$

$M_1 = M \cdot \sin 33,5$

$M_1 = 2 \sin 33,5^\circ = \cancel{1,66777\,kNm}$

$M_1 = 1,10387\,kNm$

$M_2 = 2 \cos 33,5 = 1,66777\,kNm$

$$I_{1,2} = \frac{2,9583 + 4,125}{2} \pm \sqrt{\left(\frac{2,9583 - 4,125}{2}\right)^2 + 1,375^2}$$

$$I_{1,2} = 3,5416 \pm 1,4936 \qquad I_1 = 5,035 \quad I_2 = 2,0479$$

$$tg\,2\alpha_1 = \frac{-2 \cdot 1,375}{2,9583 - 4,125} = \frac{-2,75}{-1,1667}$$

Tivadar

$$2\alpha = 180^\circ + 67^\circ \qquad \alpha = 123,5^\circ$$

a) $$\sigma_x = -\frac{M_1}{I_1}(2) + \frac{M_2}{I_2}(1) = -\frac{110,387\,kNcm}{5,035\,cm^4}(2) + \frac{166,777\,kNcm}{2,0479}(1)$$

b) $$\sigma_x = 0 \quad -\frac{110,387}{5,035}(2) + \frac{166,777}{2,0479}(1) = 0$$

(1)	0	1
(2)	0	3,71

4

$B(1,25;\ 1,92)$ $C(1,25;\ -1,08)$

$\alpha = 123,5$

$1_B = 1,25 \cos 123,5 + 1,92 \sin 123,5$

$2_B = 1,92 \cos 123,5 - 1,25 \sin 123,5 =$

o

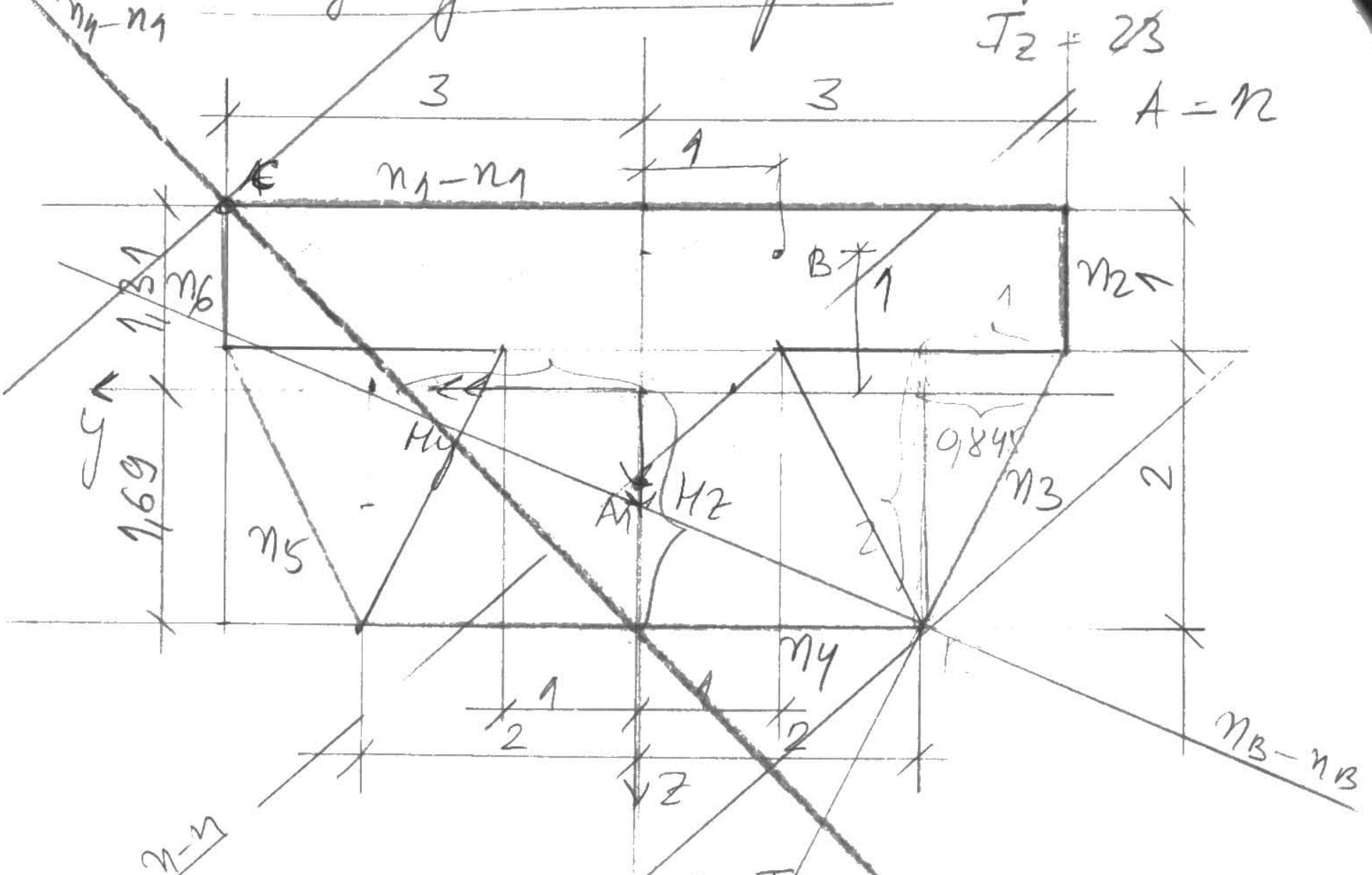
o

o

Ексцентрично напрезање $I_y = 10{,}213$

$I_z = 23$

$A = 12$



a) $N = -P$

$M_y = P \cdot 1{,}31\,cm$

$M_z = P \cdot 3\,cm$

$$i_y^2 = \frac{I_y}{A} = \frac{10{,}213}{12} = 0{,}84\ cm^2$$

$$i_z^2 = \frac{I_z}{A} = \frac{23}{12} = 1{,}91\ cm^2$$

$$\sigma_x = -\frac{P}{12cm^2} + \frac{P \cdot 1{,}31cm}{10{,}213\,cm^4}\,z - \frac{P \cdot 3cm}{23cm^4}\cdot y$$

b) n-n $\quad C(3; -1{,}31)$

$$n_y = -\frac{i_z^2}{y_c} = -\frac{1{,}91}{3} = -0{,}63$$

$$n_z = -\frac{i_y^2}{z_c} = -\frac{0{,}84}{-1{,}31} = 0{,}64$$

c) $$\sigma_x^c = -\frac{P}{12} + \frac{P \cdot 1{,}31}{10{,}213}(-1{,}31) - \frac{P \cdot 3}{23}\cdot 3 = -0{,}642\,\frac{P}{cm^2}$$

$$\tau_{max} = 0{,}32\,\frac{P}{cm^2}$$

$n_y = 1,69 \qquad n_z = 1,69$

$S(y_S; z_S) \qquad S(-1,13; -0,50)$

$$y_S = -\frac{i_z^2}{n_y} = -\frac{1,91}{1,69} = -1,13$$

$$z_S = -\frac{i_y^2}{n_z} = -\frac{0,84}{1,69} = -0,50$$

<u>$n_1 - n_1$</u> $\quad n_y = \infty \quad n_z = -1,31 \quad A_1(0; 0,64)$

$$y_1 = -\frac{i_z^2}{n_y} = -\frac{1,91}{\infty} = 0$$

$$z_1 = -\frac{i_y^2}{n_z} = -\frac{0,84}{-1,31} = 0,64$$

$A_2(0,63; 0)$

$A_6(-0,63; 0)$

<u>$n_2 - n_2$</u> $\quad n_y = -3 \quad n_z = \infty$

$$y_2 = -\frac{1,91}{-3} = 0,63$$

$$z_2 = -\frac{0,84}{\infty} = 0$$

$A_3(0,67; -0,14)$

$A_5(-0,67; -0,14)$

<u>$n_3 - n_3$</u> $\quad n_y = -2,845 \quad n_z = 5,69$

$$y_3 = -\frac{1,91}{-2,845} = 0,67 \qquad z_3 = -\frac{0,84}{5,69} = -0,14$$

<u>$n_4 - n_4$</u> $\quad n_y = \infty \quad n_z = 1,69$

Да ли тачка B припада језгру?

$B(-1; -1)$

$$n_y = -\frac{i_z^2}{y_B} = -\frac{1{,}91}{-1} = 1{,}91$$

$$n_z = -\frac{i_y^2}{z_B} = -\frac{0{,}84}{-1} = 0{,}84$$

Пошто њена н-н линија сече пресек, тачка B не припада језгру!

5. $N = ?$

$$\sigma_x^N = \frac{N}{A} = -314{,}128\ \frac{kN}{cm^2}$$

$$N = -12\,cm^2 \cdot 314{,}128\ \frac{kN}{cm^2}$$

$$N = -3769{,}53\ kN$$

$D \rightarrow S$

$\sigma_x = 2\mu \varepsilon_x + \lambda e \qquad \tau_{xy} = \mu \cdot \gamma_{xy}$

$\sigma_y = 2\mu \varepsilon_y + \lambda e \qquad \tau_{xz} = \mu \cdot \gamma_{xz}$

$\sigma_z = 2\mu \varepsilon_z + \lambda e \qquad \tau_{yz} = \mu \cdot \gamma_{yz}$

$\mu = \frac{E}{2(1+\nu)} = G \qquad \lambda = \frac{2G\nu}{1-2\nu}$

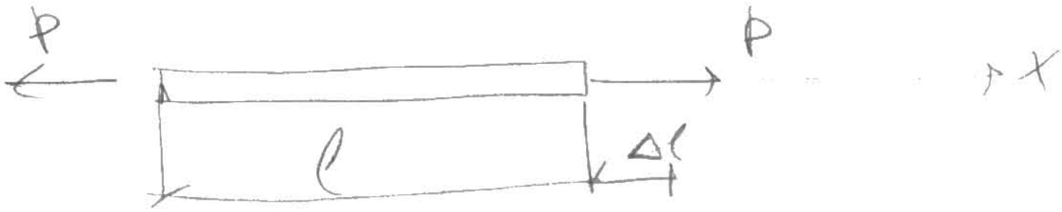
$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$

Конститутивне везе

$$D = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} 10^{-6}$$

$$D = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

$$D = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}$$



ε_x = релативно издужење = $\frac{\Delta l}{l}$

$$D = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot 10^{-6}$$

$e = 2 + 2 + 0 = 4 \cdot 10^{-6}$

$e = (2 + 2 + 0) \cdot 10^{-6}$

$E = 210\ GPa = 210\,000\ MPa$

$\nu = 0{,}25$

$$\mu = G = \frac{210\,000\ MPa}{2(1 + 0{,}25)} = 84\,000\ MPa$$

$$\lambda = \frac{2G\nu}{1 - 2\nu} = \frac{2 \cdot 84000 \cdot 0{,}25}{1 - 2 \cdot 0{,}25} = 84\,000\ MPa$$

$\sigma_x = 2 \cdot 84000 \cdot 2 \cdot 10^{-6} + 84000$

Смичући компонентални напон τ_x

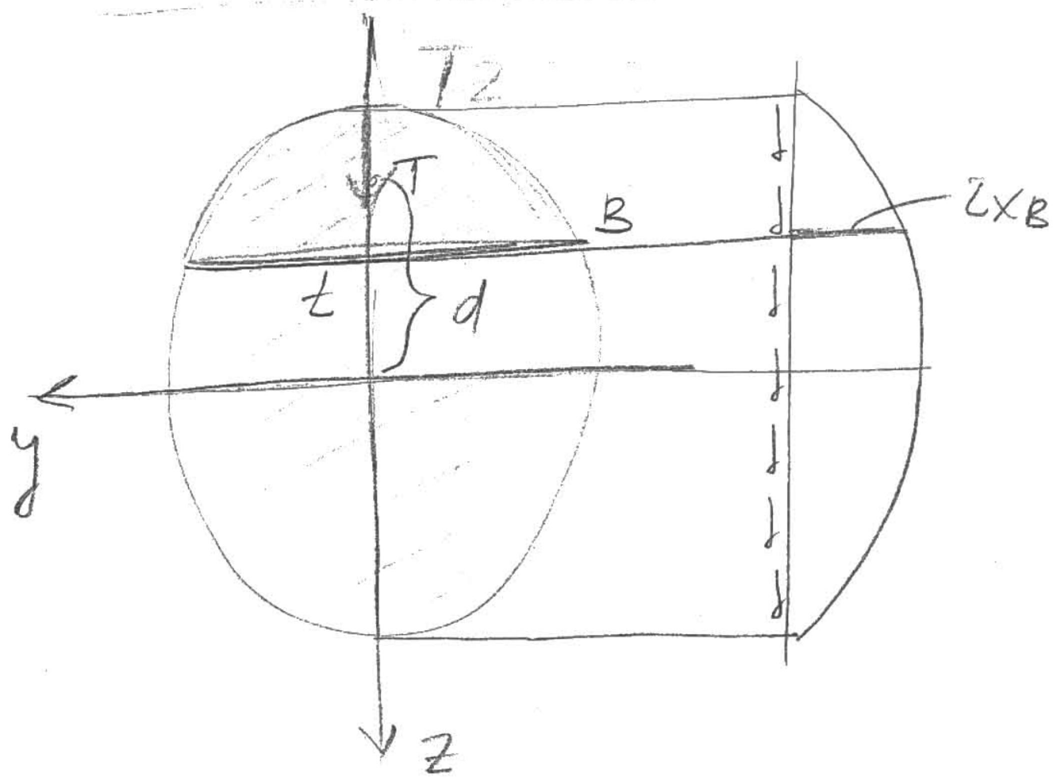
S_y – статички момент површине
t – дебљина пресека

$$\tau_x^{T_y} = \frac{T_y \cdot S_z}{I_z \cdot t}$$

$$S_z = \int y\, t\, ds$$

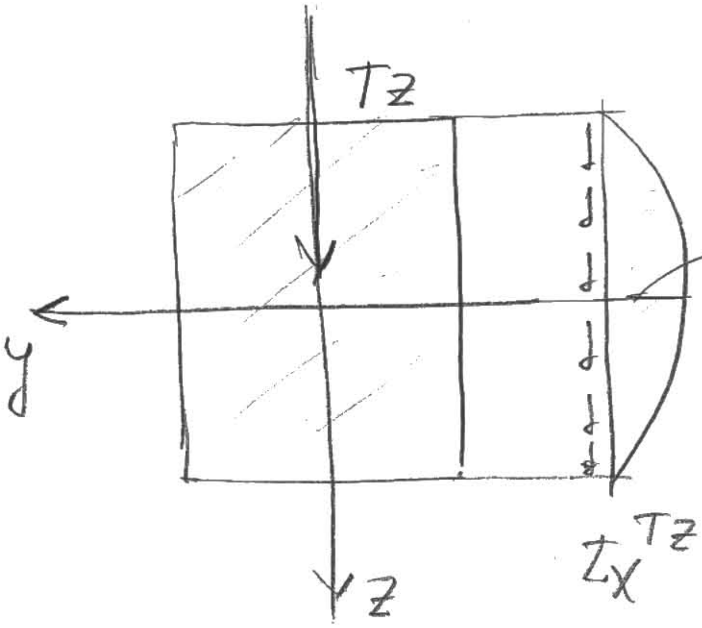
$$\tau_x^{T_z} = \frac{T_z \cdot S_y}{I_y \cdot t}$$

$$S_y = \int z\, t\, ds$$

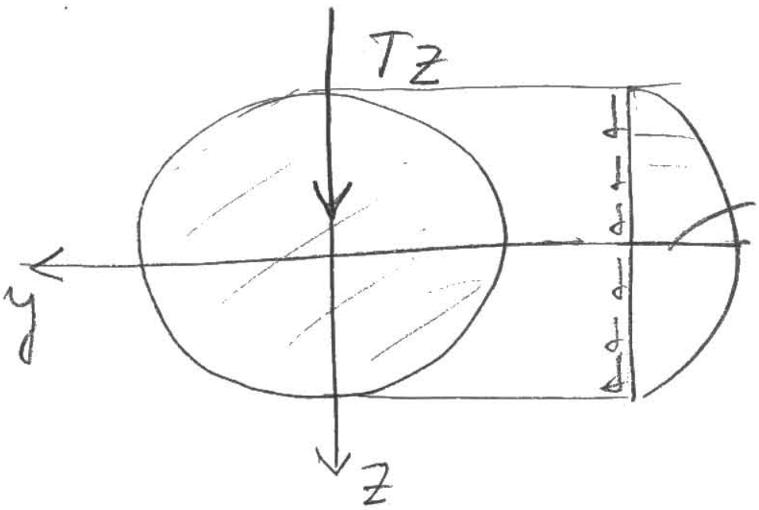


$$S_{yB} = A \cdot d_{Ty}$$

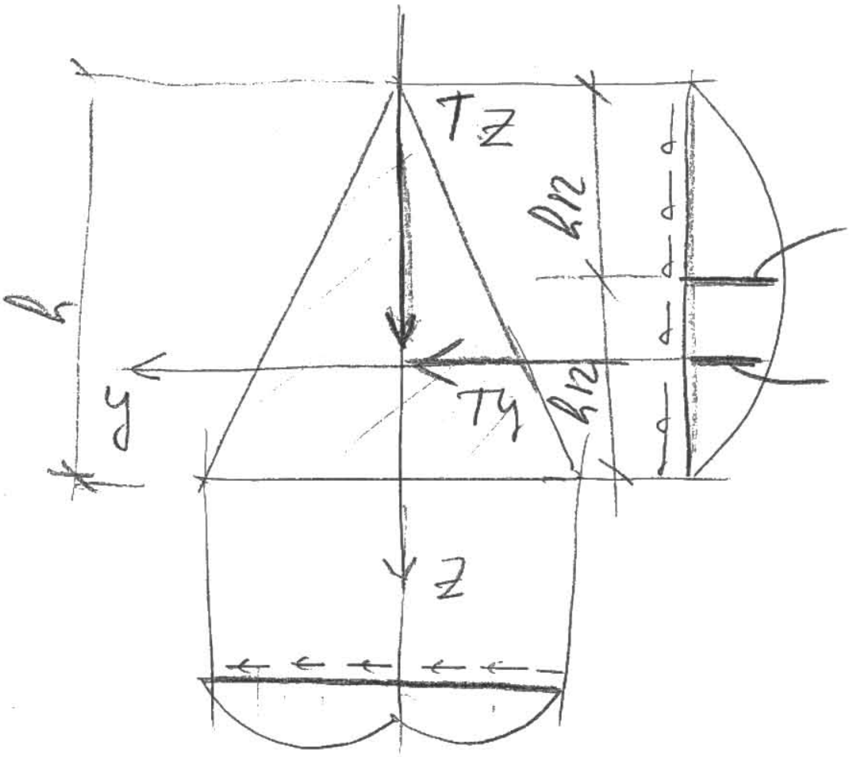
Н Јовановић



$$\tau_{x\,max} = \frac{3}{2}\,\frac{T_z}{A}$$



$$\tau_{x\,max} = \frac{4}{3}\,\frac{T_z}{A}$$



$$\tau_{x\,max} = \frac{3}{2}\,\frac{T_z}{A}$$

$$\tau_{x\,max} = \frac{4}{3}\,\frac{T_z}{A}$$

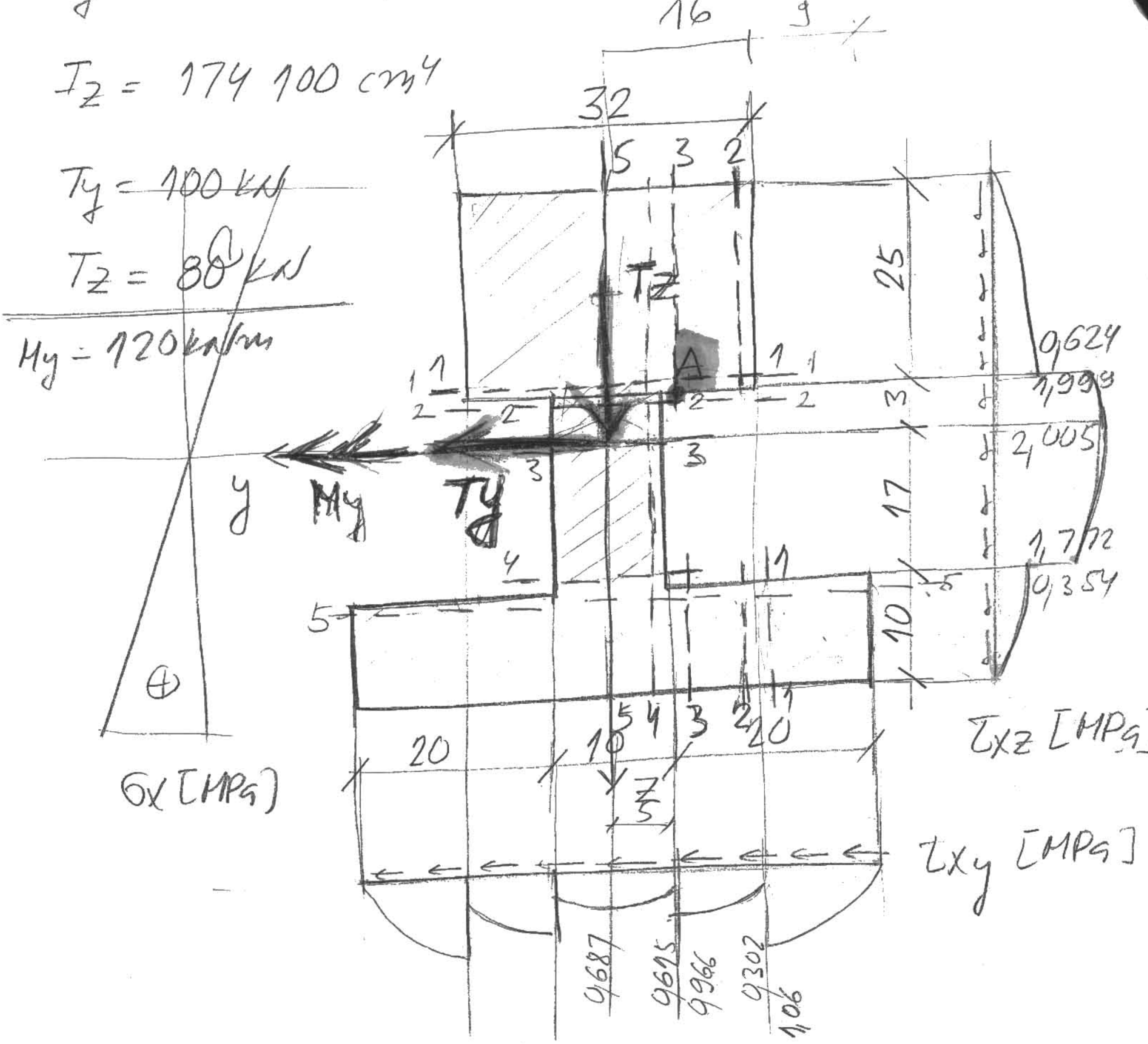
$I_y = 496\,500\ cm^4$

$I_z = 174\,100\ cm^4$

$T_y = \cancel{100\ kN}$

$T_z = 80\ kN$

$M_y = 120\ kNm$



$\tau_{xy}^A = 0{,}966\ MPa \qquad \tau_{xz}^A = 0{,}624\ MPa$

$$\sigma_x^A = \frac{M_y}{I_y} z_A = \frac{12\,000\ kNcm}{496\,500\ cm^4} \cdot (-3\ cm) = -0{,}725\ MPa$$

$$S^A = \begin{bmatrix} -0{,}725 & 0{,}966 & 0{,}624 \\ 0{,}966 & 0 & 0 \\ 0{,}624 & 0 & 0 \end{bmatrix} MPa$$

$$\tau_x^{T_z} = \frac{T_z \cdot S_y}{I_y \cdot t} \qquad I_y = 496\,500\ cm^4$$

$$S_{y1} = S_{y2} = A \cdot d_{1y} = 32 \cdot 25 \cdot 15{,}5 = 12\,400\ cm^3$$

$$S_{y3} = 12\,400 + 10 \cdot 3 \cdot 1{,}5 = 12\,445\ cm^3$$

$$S_{y4} = S_{y5} = 50 \cdot 10 \cdot 22 = 11\,000\ cm^3$$

$$\tau_{x1} = \frac{80\ kN \cdot 12\,400\ cm^3}{496\,500\ cm^4 \cdot 32\ cm} = 0{,}624\ MPa$$

$$\tau_{x2} = \frac{80 \cdot 12\,400}{496\,500 \cdot 10} = 1{,}998\ MPa$$

$$\tau_{x3} = \frac{80 \cdot 12\,445}{496\,500 \cdot 10} = 2{,}005\ MPa$$

$$\tau_{x4} = \frac{80 \cdot 11\,000}{496\,500 \cdot 10} = 1{,}772\ MPa$$

$$\tau_{x5} = \frac{80 \cdot 11\,000}{496\,500 \cdot 50} = 0{,}354\ MPa$$

$$\tau_x^{T_y} = \frac{T_y \cdot S_z}{I_z \cdot t}$$

$$S_{z1} = S_{z2} = 9 \cdot 10 \cdot 20{,}5 = 1845\ cm^3$$

$$S_{z3} = S_{z4} = 1845 + 11 \cdot 35 \cdot 10{,}5 = 5887{,}5$$

$$S_{z5} = 5887{,}5 + 5 \cdot 55 \cdot 2{,}5 = 6575$$

$$\tau_{x1} = \frac{100 \cdot 1845\ cm^3}{174\ 100 \cdot 10} = 1{,}06\ MPa$$

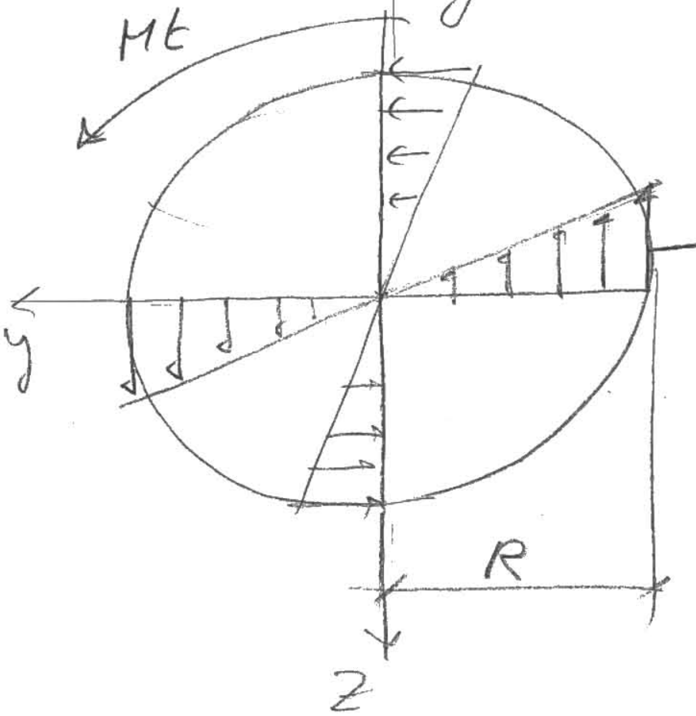
$$\tau_{x2} = \frac{100 \cdot 1845}{174100 \cdot 35} = 0{,}302\ MPa$$

$$\tau_{x3} = \frac{100 \cdot 5887{,}5}{174\ 100 \cdot 35} = 0{,}966\ MPa$$

$$\tau_{x4} = \frac{100 \cdot 5887{,}5}{174\ 100 \cdot 55} = 0{,}615\ MPa$$

$$\tau_{x5} = \frac{100 \cdot 6575}{174100 \cdot 55} = 0{,}6866\ MPa$$

τ_x од момента торзије

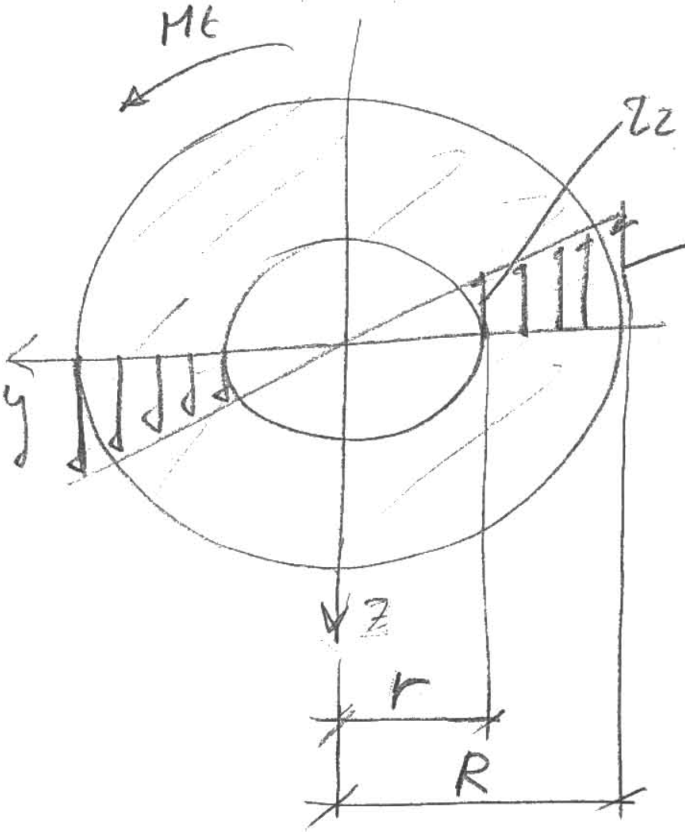


$$M_t \oplus$$

$$M_t \ominus$$

$$\tau_x^{max} = \frac{M_t}{I_t} \cdot R$$

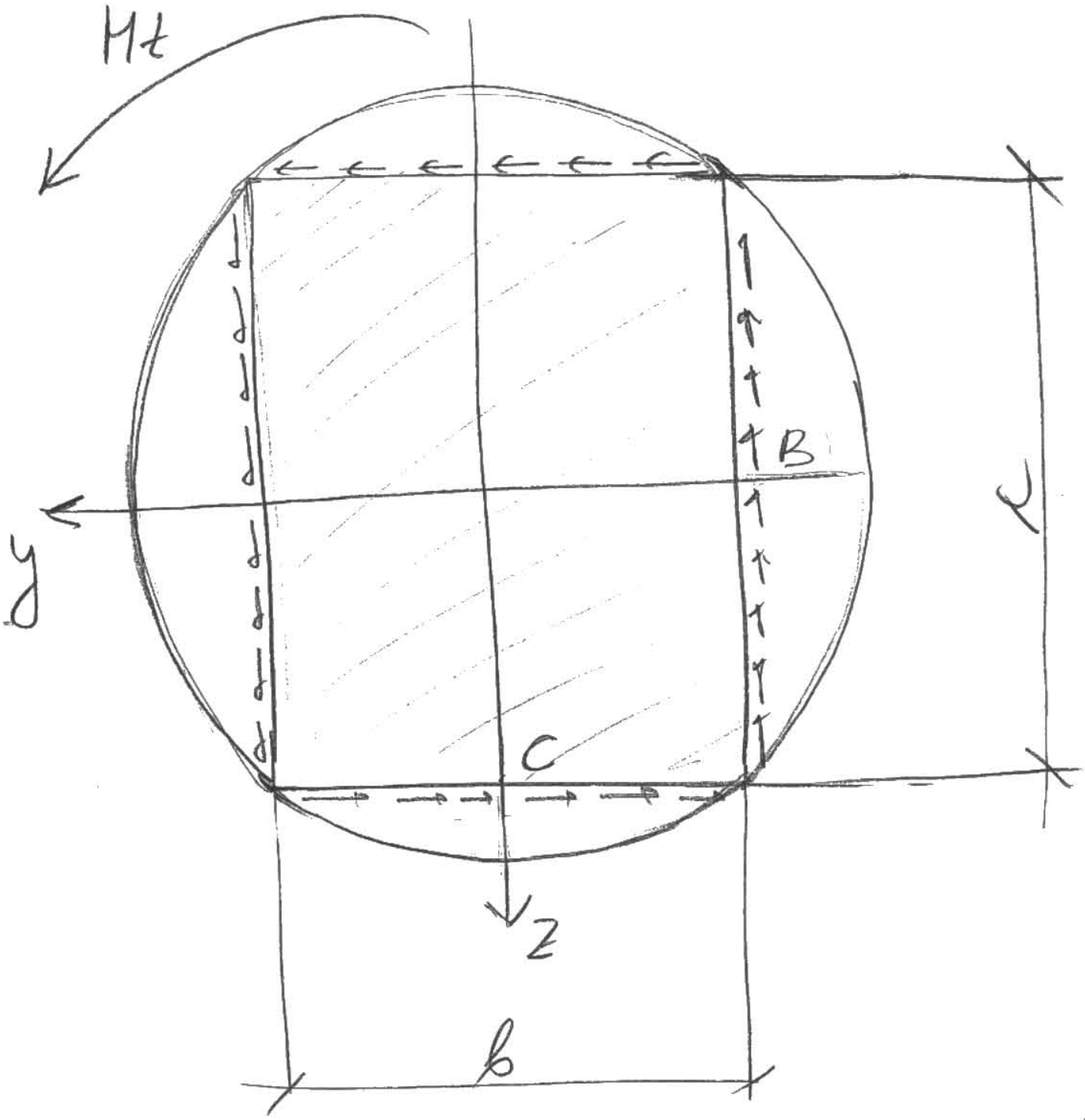
$$I_t = \frac{R^4 \pi}{2}$$



$$\tau_1 = \frac{M_t}{I_t} \cdot R$$

$$\tau_2 = \frac{M_t}{I_t} \cdot r$$

$$I_t = \frac{R^4 \pi}{2} - \frac{r^4 \pi}{2}$$



$$\tau_x^B = \frac{Mt}{W_{tB}} \qquad W_{tB} = \beta \cdot b^2 c$$

$$\tau_x^C = \frac{Mt}{W_{tC}} \qquad W_{tC} = \gamma \cdot b^2 c$$

$$I_t = \alpha \cdot b^3 c$$

$\mu = \frac{c}{b}$

$\Downarrow$

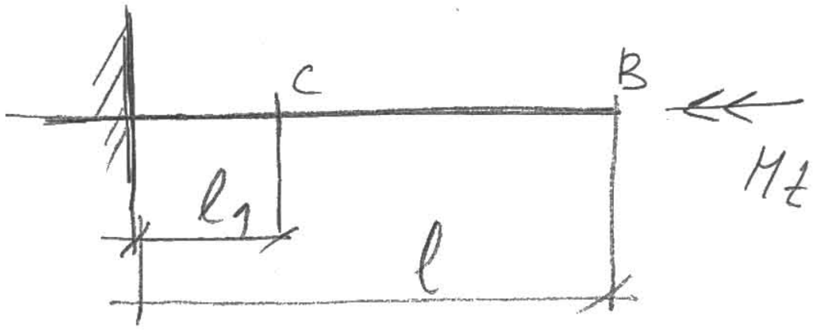
таблице

α

β

γ

угао увијања



$$\varphi_B = \frac{M_t \cdot l}{G I_t}$$

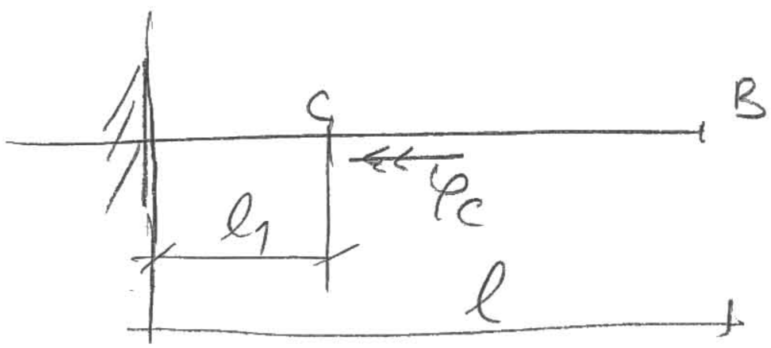
G = модул смицања (клизања)

$$G = \frac{E}{2(1+\nu)}$$

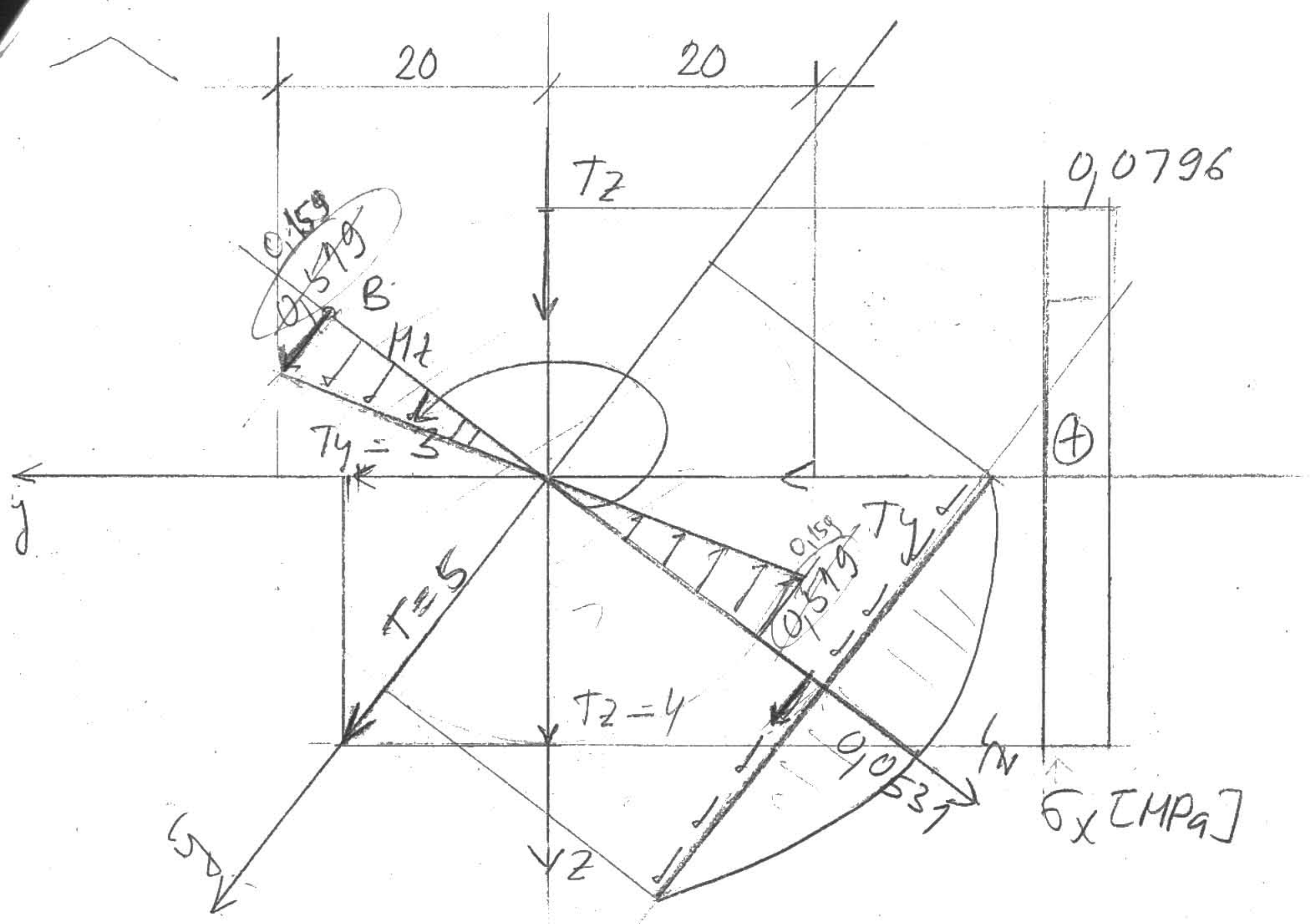
E = модул еластичности

ν = Поасонов коеф

$$\varphi_C = \frac{M_t \cdot l_1}{G \cdot I_t}$$

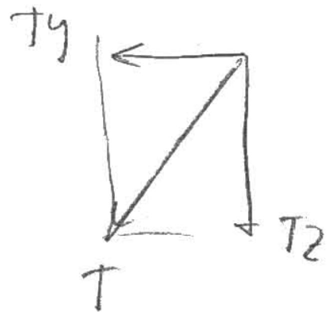


$$\varphi_B = \varphi_C = \frac{M_t l_1}{G I_t}$$



$$\begin{cases} N = 10\,kN \\ T_y = 3\,kN \\ T_z = 4\,kN \\ M_t = 2\,kNm \end{cases}$$

$A = 20^2\pi = 400\pi\ cm^2$



a) $\sigma_x = \frac{N}{A} = \frac{10\,kN}{400\pi\,cm^2} = 0{,}0796\ MPa$

$\tau_x^T = \frac{4}{3}\frac{T}{A} = \frac{4 \cdot 5\,kN}{3 \cdot 400\pi\,cm^2} = 0{,}0531\ MPa$

$\tau_x^{Mt} = \frac{M_t}{I_t} \cdot R \qquad I_t = \frac{R^4\pi}{2} = \frac{20^4\pi}{2}cm^4 = 251\ 327{,}41\,cm^4$

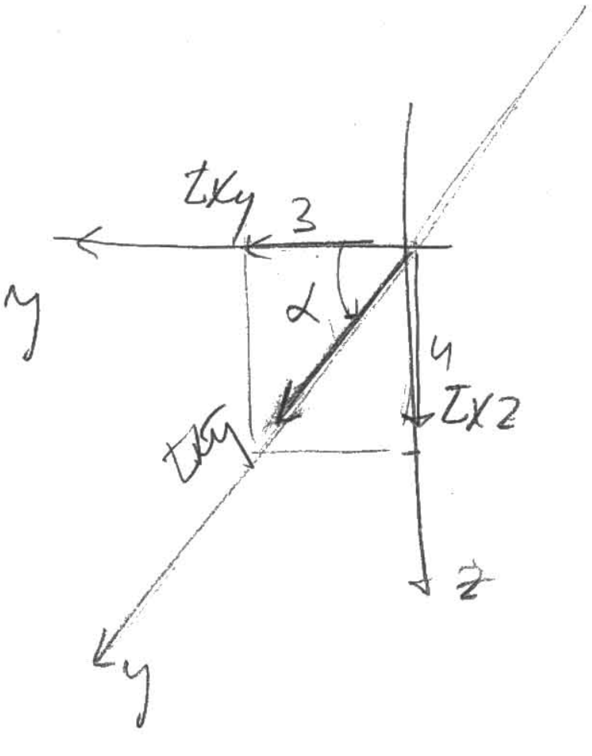
$\tau_x^{Mt} = \frac{200\,kNcm}{251\ 327{,}41\,cm^4} \cdot 20\,cm = 0{,}159\ MPa$

$\sigma_x^B = 0{,}0796\ MPa$

$\tau_{x\bar{y}} = 0{,}0531 + 0{,}159 = \underline{0{,}2121}\ MPa$

$$S_{x\bar{y}\bar{z}} = \begin{bmatrix} 0{,}0796 & 0{,}2121 & 0 \\ 0{,}2121 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$

Мохров круг



$\cos\alpha = 0{,}6$

$\sin\alpha = 0{,}8$

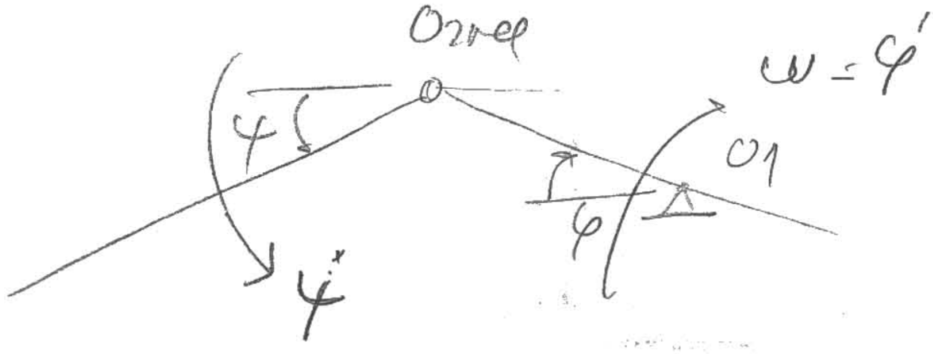
$\tau_{xy} = \tau_{x\bar{y}} \cdot 0{,}6 = 0{,}2121 \cdot 0{,}6$

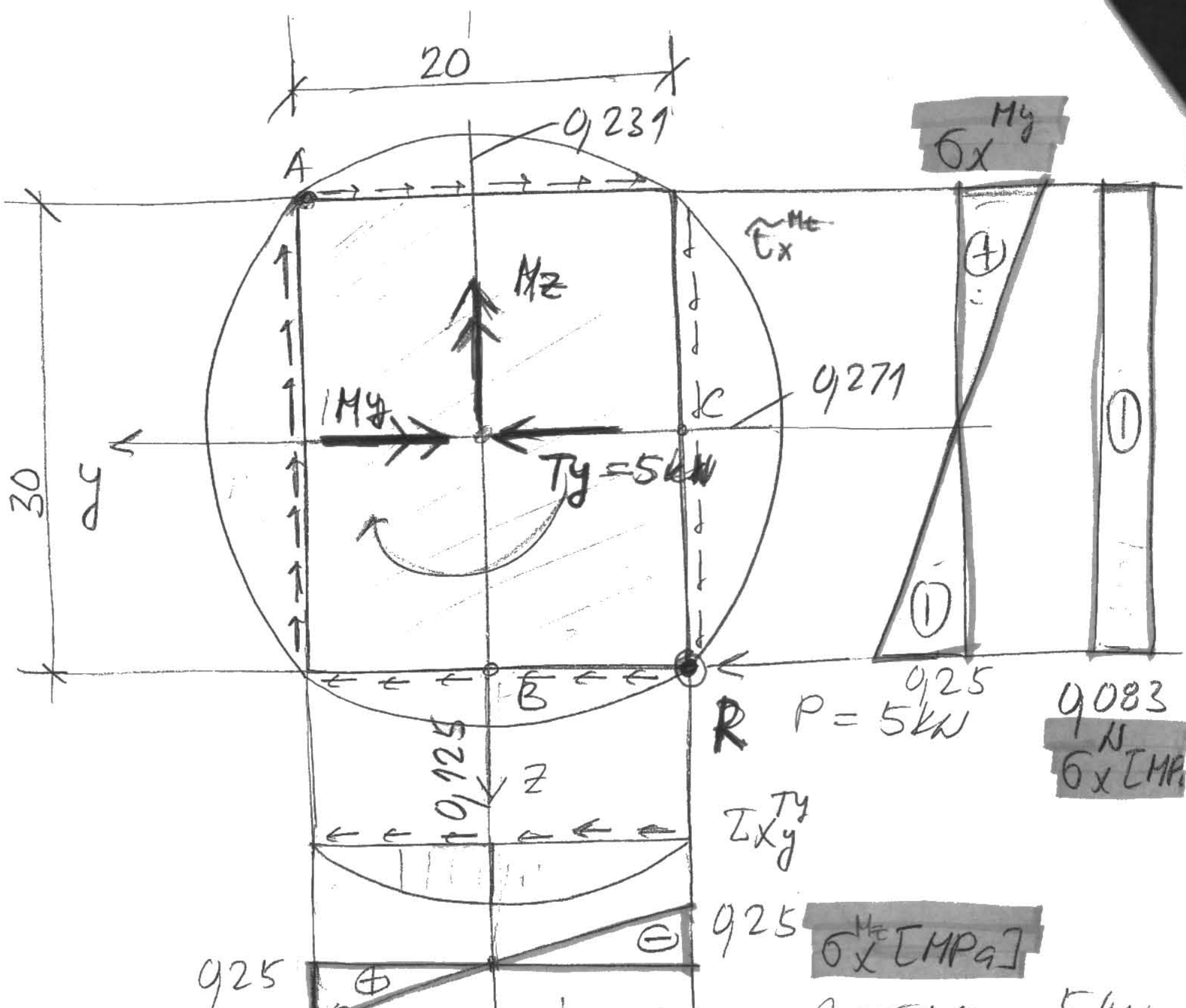
$\tau_{xz} = \tau_{x\bar{y}} \cdot 0{,}8 = 0{,}2121 \cdot 0{,}8$

$\tau_{xy} = 0{,}12726\ MPa$

$\tau_{xz} = 0{,}1696\ MPa$

$$S_{xyz} = \begin{bmatrix} 0{,}0796 & 0{,}127 & 0{,}170 \\ 0{,}127 & 0 & 0 \\ 0{,}170 & 0 & 0 \end{bmatrix} MPa$$


Orel
ω = φ'
O1
φ
φ
φ̇



у тачки R делују сила притиска 5kN
и сила P

a) A, B, C

$N = -5\,kN$

$T_y = 5\,kN$

$M_y = 5\,kN \cdot 15\,cm = 75\,kNcm$

$M_z = 5\,kN \cdot 10\,cm = 50\,kNcm$

$M_t = 5\,kN \cdot 15\,cm = 75\,kNcm$

$$\tau_x^{T_y} = \frac{3}{2}\frac{T_y}{A} = \frac{3}{2}\cdot\frac{5\,kN}{600\,cm^2} = 0{,}125\ MPa$$

$$\tau_x^{B} = \frac{M_t}{W_{tB}} \qquad \mu = \frac{c}{b} = \frac{30}{20} = \underline{1{,}5}$$

$$W_{tB} = \beta \cdot b^2 c = 0{,}231 \cdot 20^2 \cdot 30\,cm^3 = \underline{2772\,cm^3}$$

$$W_{tC} = \beta\, b^2 c = 0{,}270 \cdot 20^2 \cdot 30\,cm^3 = \underline{\underline{3240\,cm^3}}$$

$$\tau_x^{B} = \frac{75\,kNcm}{2772\,cm^3} = 0{,}271\ MPa$$

$$\tau_x^{C} = \frac{75\,kNcm}{3240\,cm^3} = 0{,}231\ MPa$$

$$\sigma_x^{N} = \frac{N}{A} = \frac{-5\,kN}{600\,cm^2} = -0{,}083\ MPa$$

$$\sigma_x^{M_y} = -\frac{M_y}{I_y}z = \mp\frac{75\,kNcm}{\frac{20\cdot 30^3}{12}}\cdot 15\,cm = 0{,}25\ MPa$$

$$\sigma_x^{M_z} = \frac{M_z}{I_z}\cdot y = \frac{50\,kNcm}{\frac{30\cdot 20^3}{12}}\cdot 10\,cm = 0{,}25\ MPa$$

(12)

$$\sigma_x^A = 0{,}25 + 0{,}25 - 0{,}083 = 0{,}417 \text{ MPa}$$

$$S^A = \begin{bmatrix} 0{,}417 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

$$\sigma_x^B = -0{,}25 - 0{,}083 = -0{,}338$$

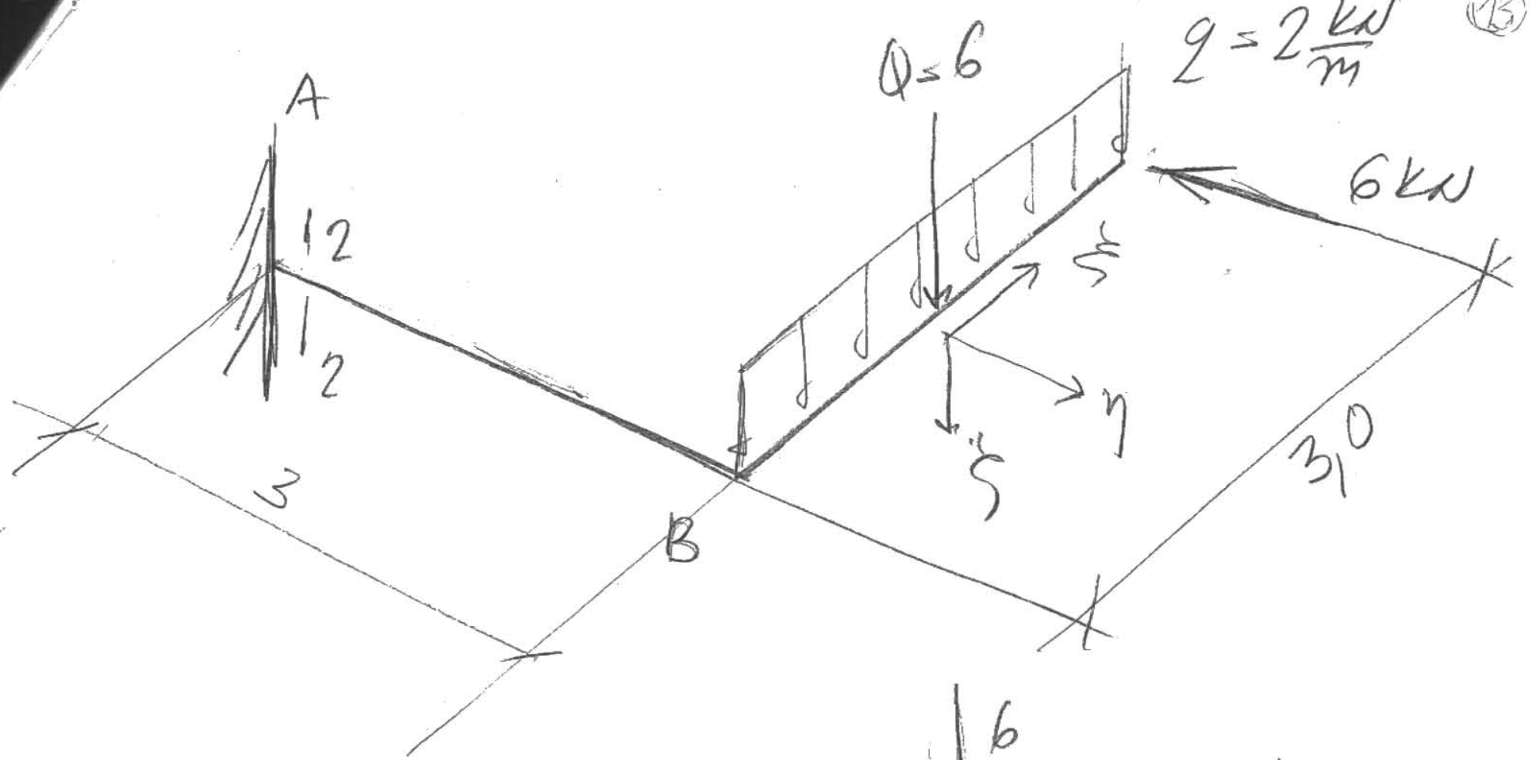
$$\tau_{xy} = 0{,}125 + 0{,}231 = 0{,}356$$

$$S^B = \begin{bmatrix} -0{,}338 & 0{,}356 & 0 \\ 0{,}356 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa} \rightarrow \boxed{\begin{array}{l} \sigma_{1,2}^B = \\ tg\, 2\alpha_1 = \end{array}}$$

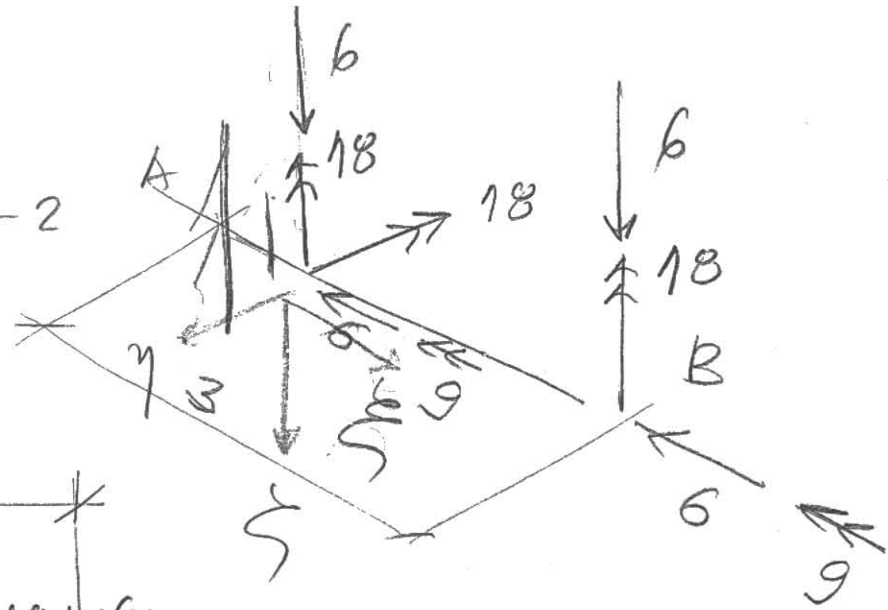
$$\sigma_x^C = -0{,}083 - 0{,}25 = -0{,}338 \text{ MPa}$$

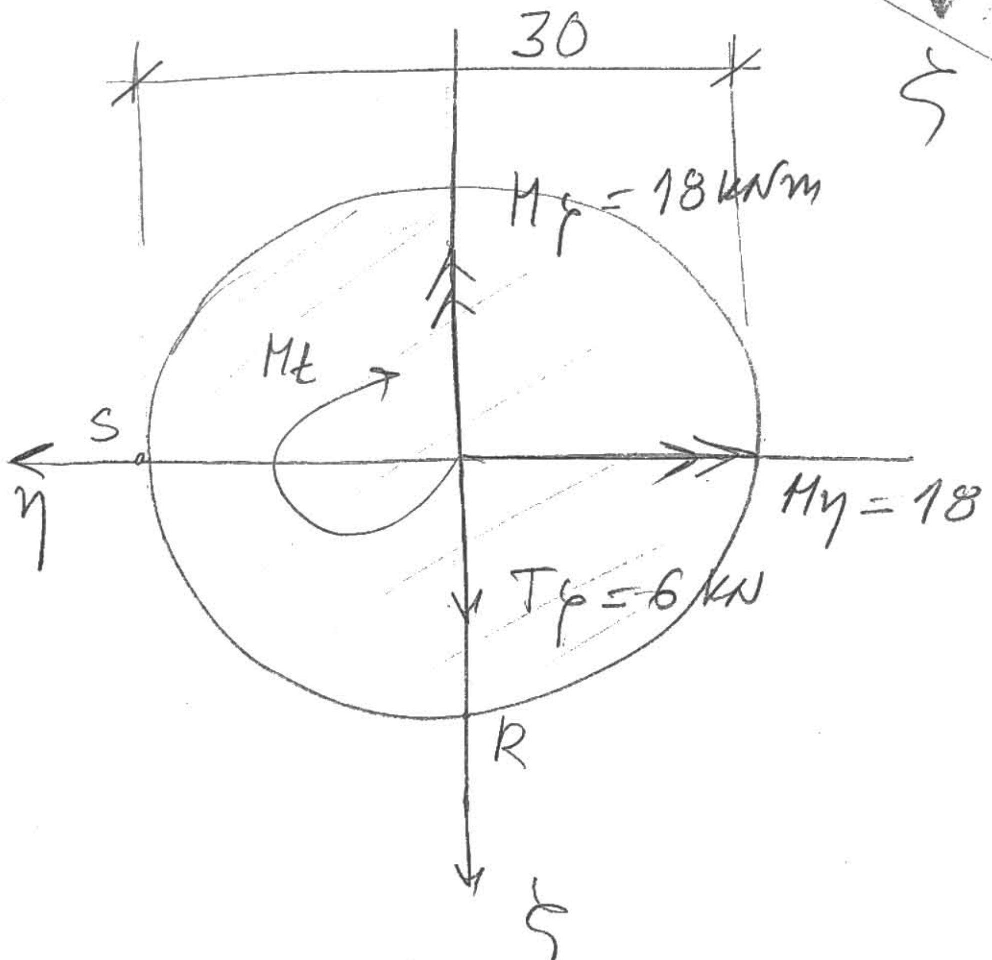
$$\tau_{xz}^C = 0{,}271$$

$$S^C = \begin{bmatrix} -0{,}338 & 0{,}271 & 0 \\ 0{,}271 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$



c) на пресеку 2-2





N = -6 kN

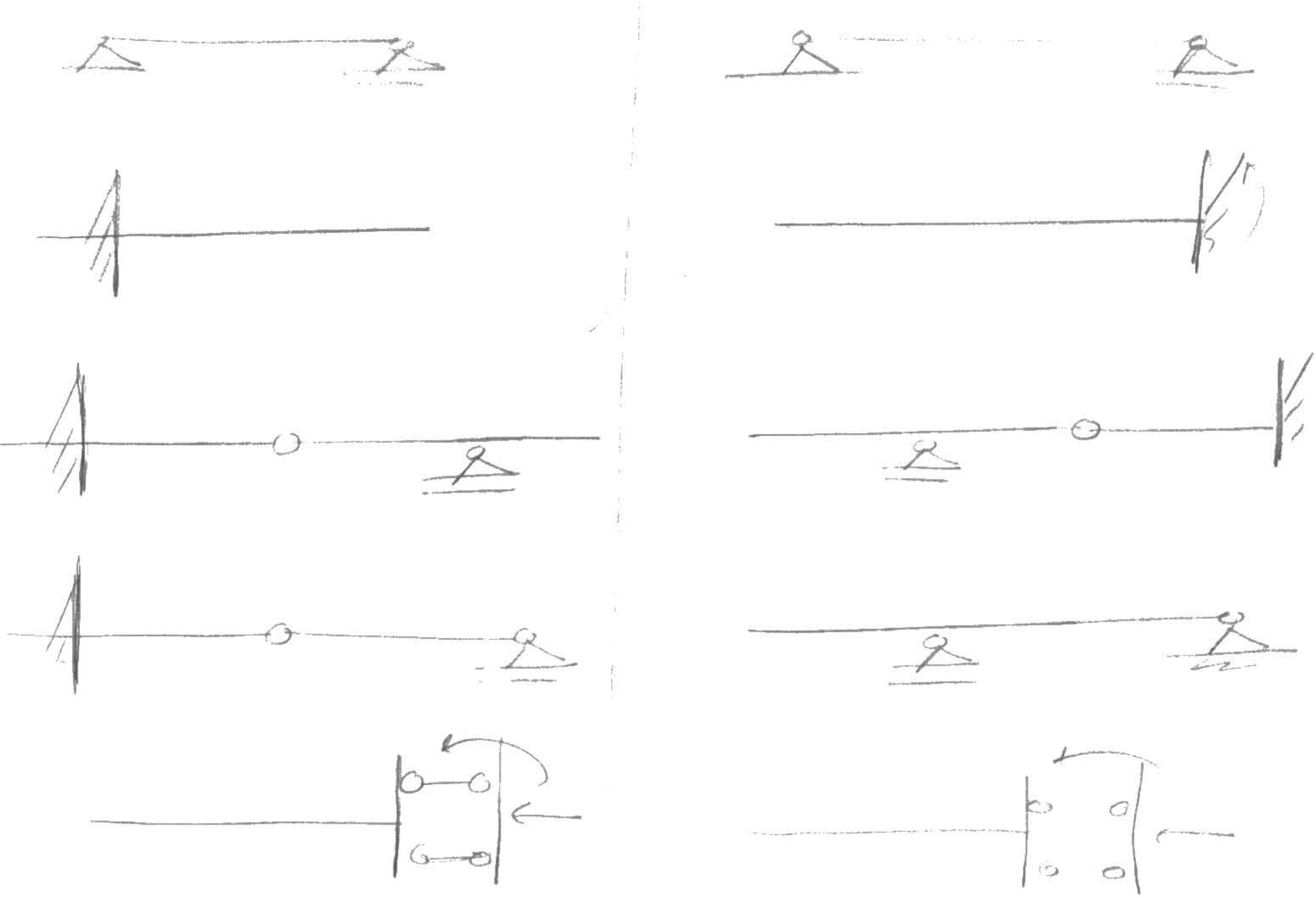
Mt = -9 kNm

Мор - Максвелова аналогија

угиб $w[m] = \dfrac{\overline{M}\,[kNm^3]}{EI\,[kNm^2]}$ H. Jovanović

нагиб $\varphi[rad] = \dfrac{\overline{T}\,[kNm^2]}{EI\,[kNm^2]}$

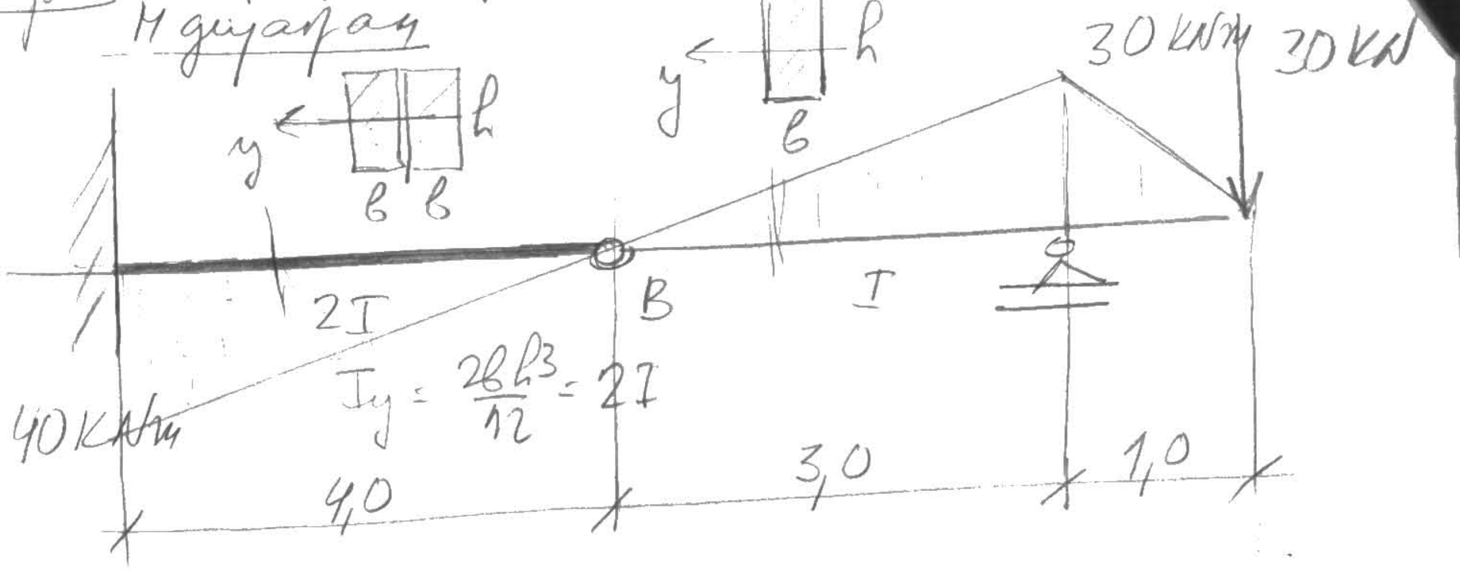
стварни фиктивни носачи



Стварни Пример

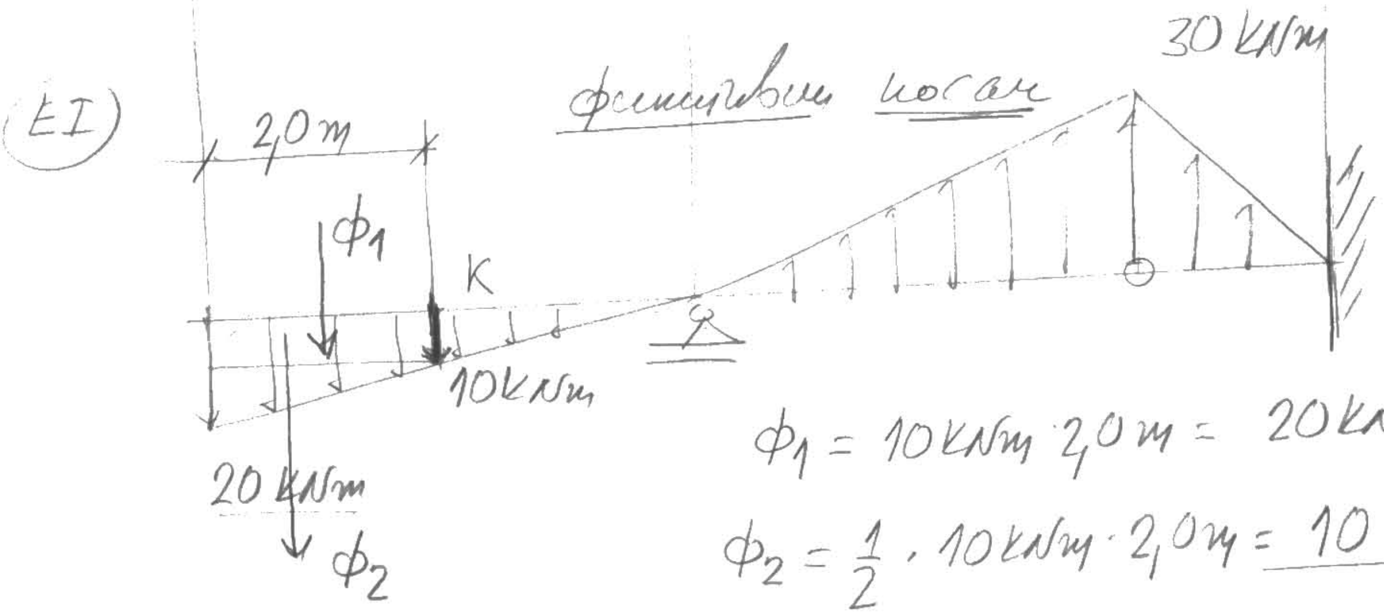
$I = I_y = \frac{bh^3}{12}$

М дијаграм



(EI)

фиктивни носач



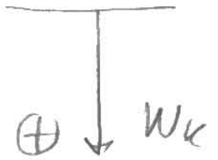
$\Phi_1 = 10\,kNm \cdot 2{,}0\,m = 20\,kNm^2$

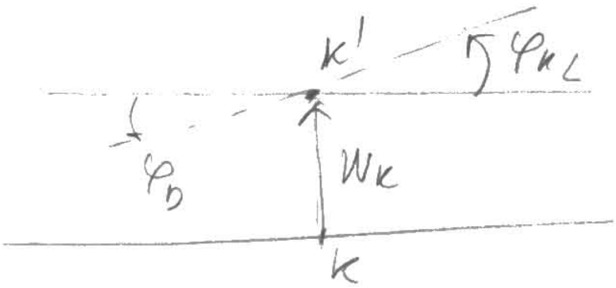
$\Phi_2 = \frac{1}{2} \cdot 10\,kNm \cdot 2{,}0\,m = \underline{10\,kNm^2}$

$$w_k = \frac{1}{EI}\bar{M}_k = \frac{1}{EI}\left[-20\,kNm^2 \cdot 1m - 10\,kNm^2 \cdot \frac{2}{3} \cdot 2m\right]$$

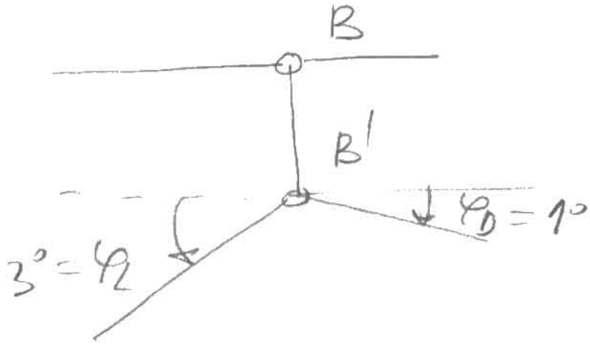
$$w_k = \frac{1}{EI}\left[-33{,}333\,kNm^3\right]$$

$$\varphi_k = \frac{1}{EI}\bar{T}_k = \frac{1}{EI}\left[-30\,kNm^2\right]$$

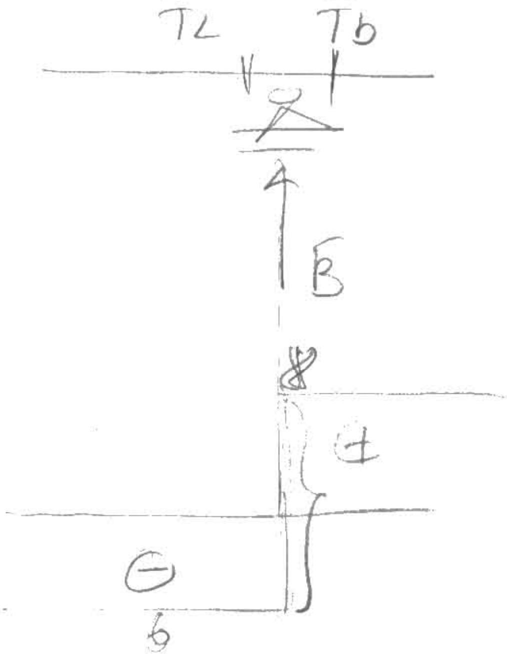




Згод B



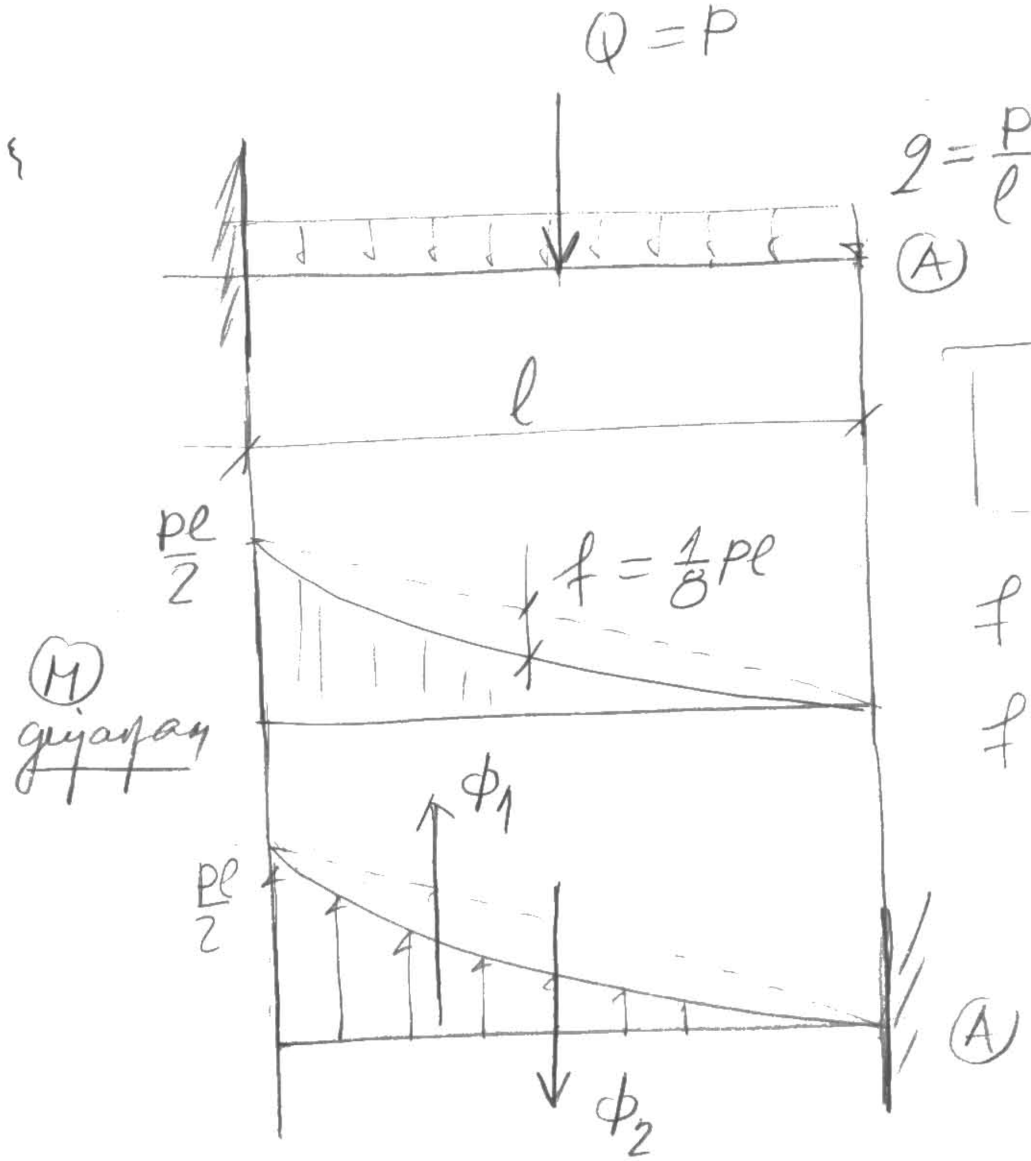
$$\Delta\varphi_B = \varphi_D - \varphi_L = 1^\circ - (-3^\circ) = \underline{4^\circ}$$



$$\underbrace{\Delta\varphi = \frac{\bar{B}}{EI}}$$

Промена угла у згоду B

$$\Delta T = 8 - (-6) = 8 + 6 = \underline{14}$$



$$\boxed{P = \frac{2}{3} l f}$$

$$f = \frac{1}{8} q l^2$$

$$f = \frac{1}{8} \frac{P}{l} \cdot l^2 = \frac{1}{8} P l$$

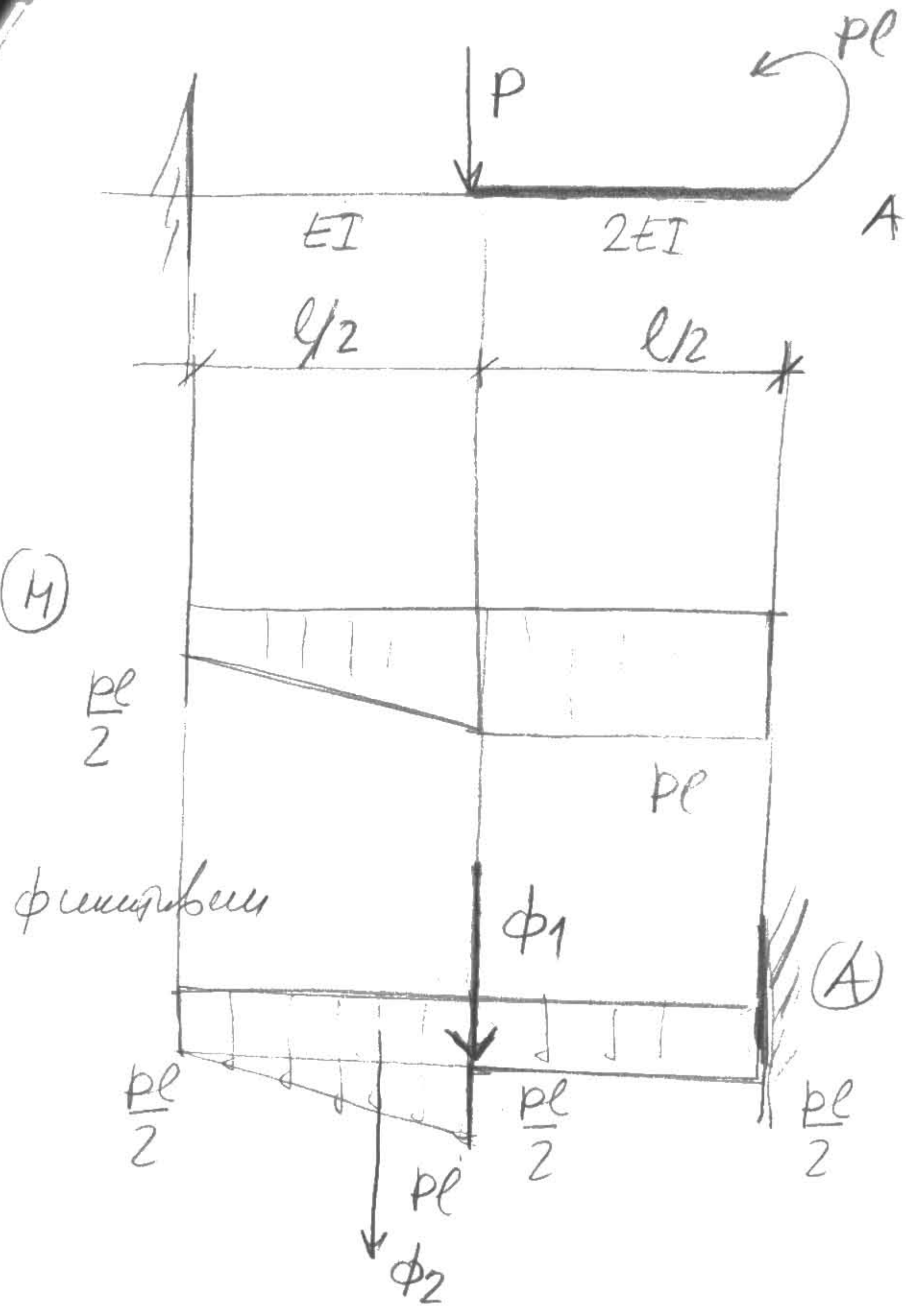
$$\Phi_1 = \frac{1}{2} l \cdot \frac{Pl}{2} = \frac{1}{4} P l^2$$

$$\Phi_2 = \frac{2}{3} l f = \frac{2}{3} l \cdot \frac{1}{8} P l = \frac{1}{12} P l^2$$

$$w_A = \frac{1}{EI}\left[\frac{1}{4} P l^2 \cdot \frac{2}{3} l - \frac{1}{12} P l^2 \cdot \frac{1}{2} l\right] = \frac{1}{EI}\left[\frac{2}{12} P l^3 - \frac{1}{24} P l^3\right]$$

$$w_A = \frac{1}{EI} \frac{1}{8} P l^3$$

$$\varphi_A = \frac{1}{EI}\left[\Phi_1 - \Phi_2\right] = \frac{1}{EI}\left[\frac{1}{4} P l^2 - \frac{1}{12} P l^2\right] = \frac{1}{EI} \cdot \frac{1}{6} P l^2$$



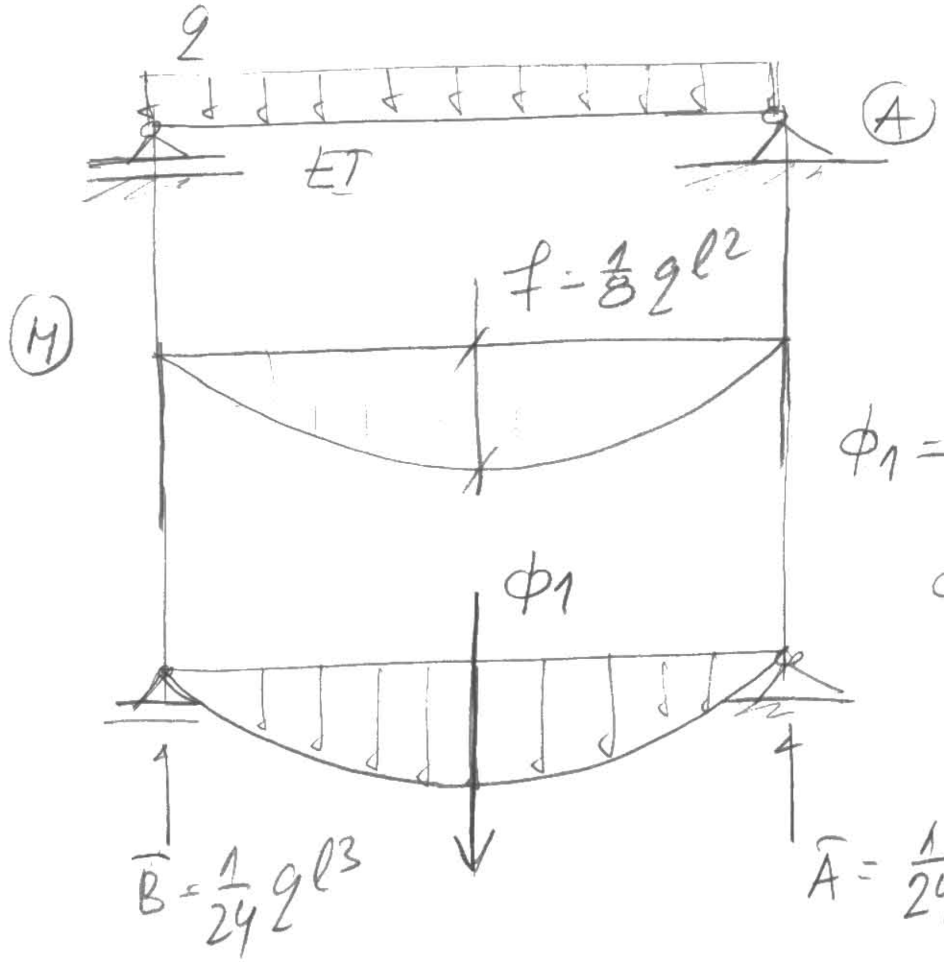
$$\Phi_1 = l \cdot \frac{Pl}{2} = \frac{1}{2} Pl^2$$

$$\Phi_2 = \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{Pl}{2} = \frac{1}{8} Pl^2$$

$$w_A = \frac{1}{EI}\left[-\Phi_1 \cdot \frac{l}{2} - \Phi_2 \left(\frac{l}{2} + \frac{1}{3}\,\frac{l}{2} \right) \right]$$

$$\varphi_A = \frac{1}{EI}\left[-\Phi_1 - \Phi_2 \right]$$

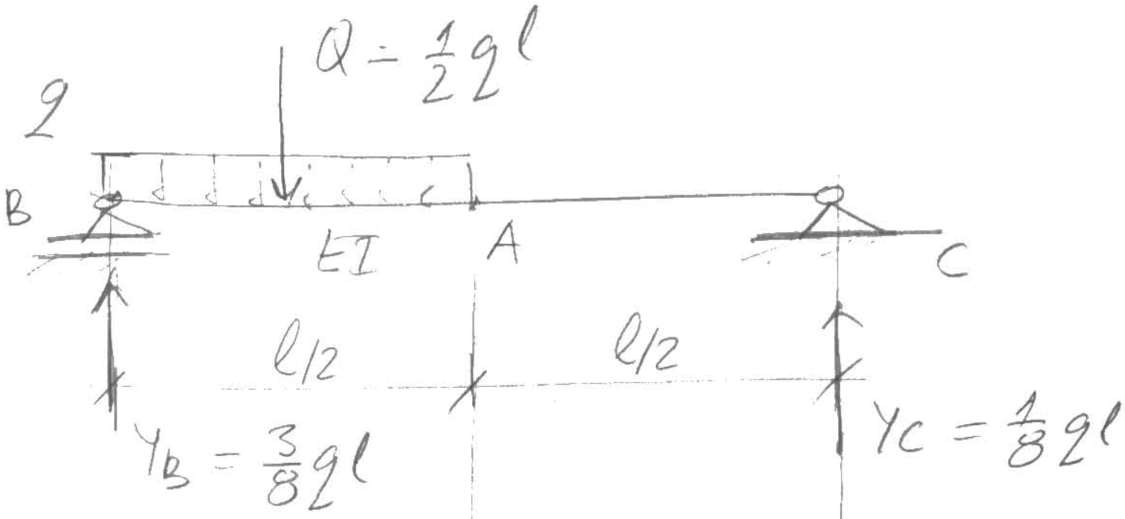
$$w_A = 0$$



$$\Phi_1 = \frac{2}{3} l f = \frac{2}{3} l \cdot \frac{1}{8} q l^2$$

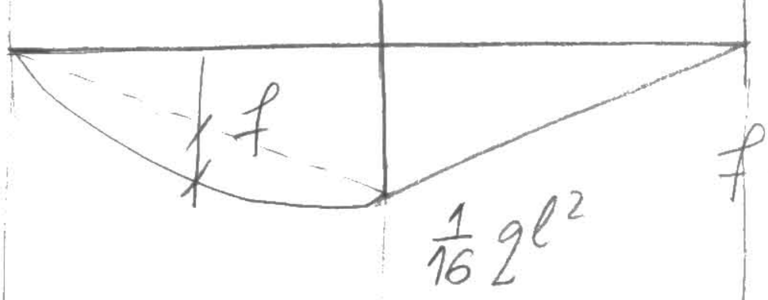
$$\Phi_1 = \frac{1}{12} q l^3$$

$$\varphi_A = \frac{1}{EI} \bar{T}_A = \frac{1}{EI}\left[-\frac{1}{24} q l^3\right]$$

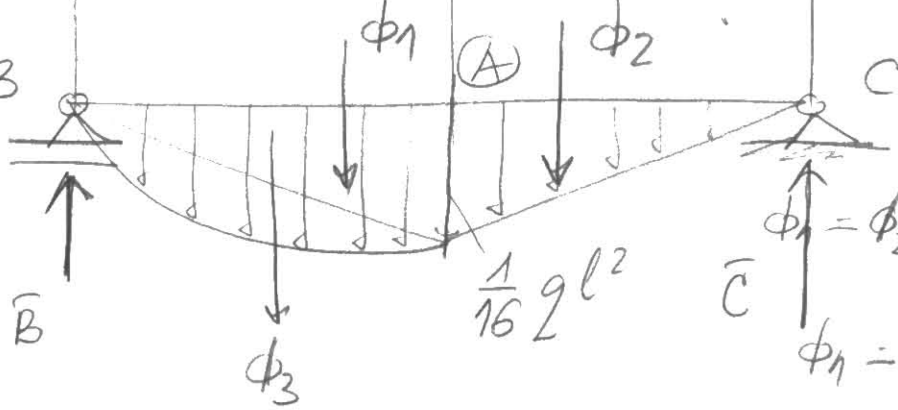


$\sum M_B = 0$ $\quad \frac{1}{2} q l \cdot \frac{l}{4} - Y_C \cdot l = 0$ $\quad \underline{Y_C = \frac{1}{8} q l}$

<u>M dijagram</u>



$f = \frac{1}{8} \cdot q \cdot \left(\frac{l}{2}\right)^2 = \frac{1}{32} q l^2$



$\Phi_1 = \Phi_2 = \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{1}{16} q l^2 = \frac{1}{64} q l^3$

$\Phi_1 = \Phi_2 = \frac{1}{64} q l^3$

$\Phi_3 = \frac{\cancel{2}}{3} \cdot \frac{l}{\cancel{2}} \cdot \frac{1}{32} q l^2$

$\Phi_3 = \frac{1}{96} q l^3$

$\sum M_B = 0$

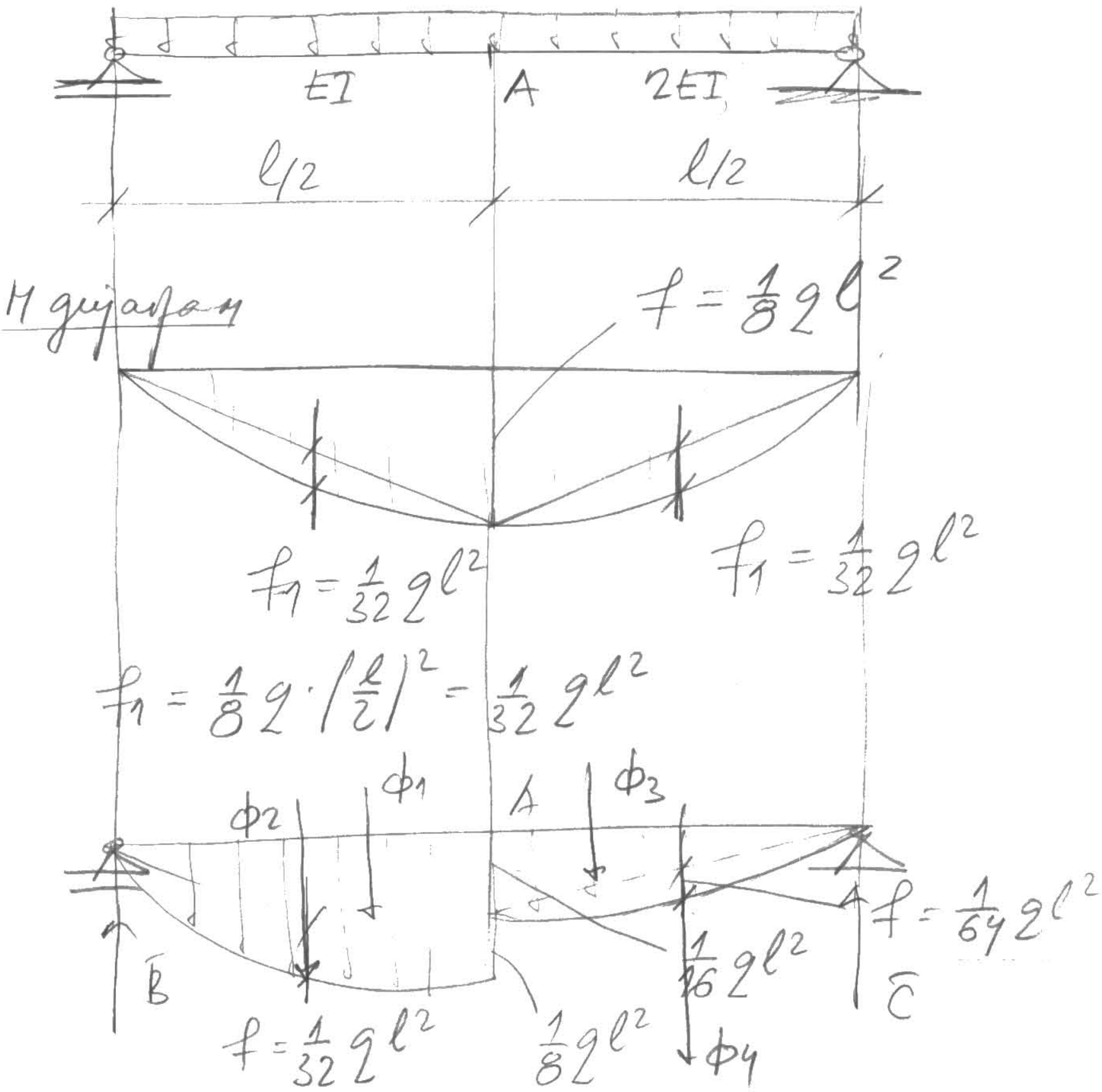
$-\bar{C} \cdot l + \frac{1}{64} q l^3 \left(\frac{2}{3} \cdot \frac{1}{2} l\right) + \frac{1}{64} q l^3 \left(\frac{l}{2} + \frac{1}{3} \frac{l}{2}\right) +$

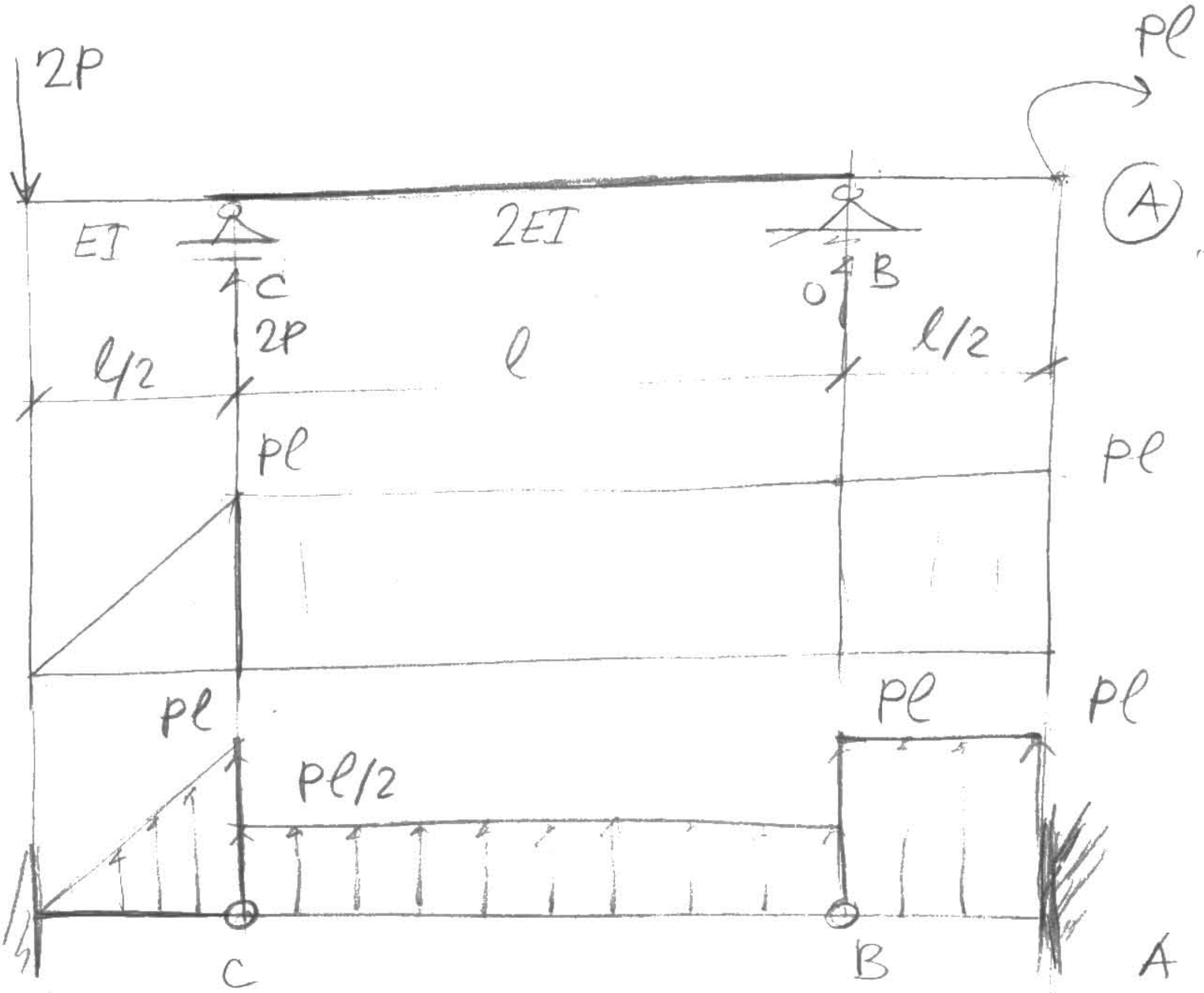
$+ \frac{1}{32} q l^2 \cdot \frac{l}{4} = 0 \qquad \underline{\bar{C} = \frac{3}{128} q l^3}$

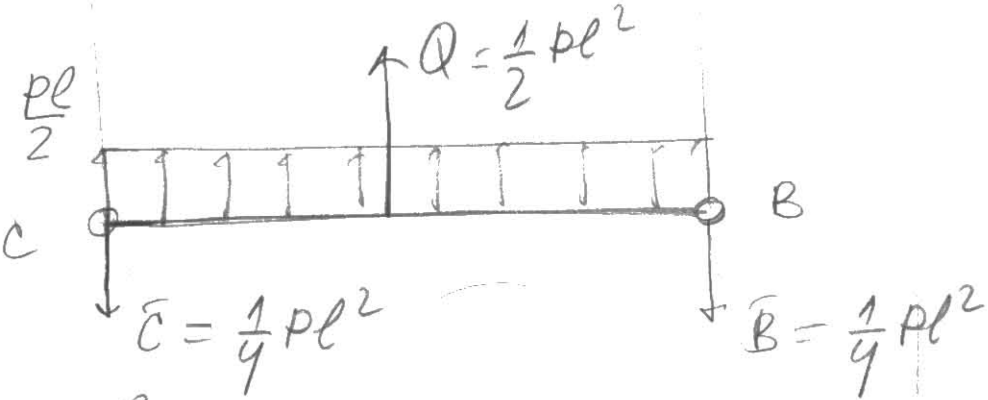
$w_A = \frac{1}{EI} \left[\frac{3}{128} q l^3 \cdot \frac{l}{2} - \frac{1}{64} q l^3 \cdot \frac{l}{6} \right]$

$\varphi_A = \frac{1}{EI} \left[\frac{1}{64} q l^3 - \frac{3}{128} q l^3 \right]$

2


EI
A
2EI
l/2
l/2
M dijagram
f = 1/8 ql²
f₁ = 1/32 ql²
f₁ = 1/32 ql²
f₁ = 1/8 q·(l/2)² = 1/32 ql²
Φ2
Φ1
A
Φ3
B̄
C̄
f = 1/64 ql²
1/16 ql²
f = 1/32 ql²
1/8 ql²
Φ4

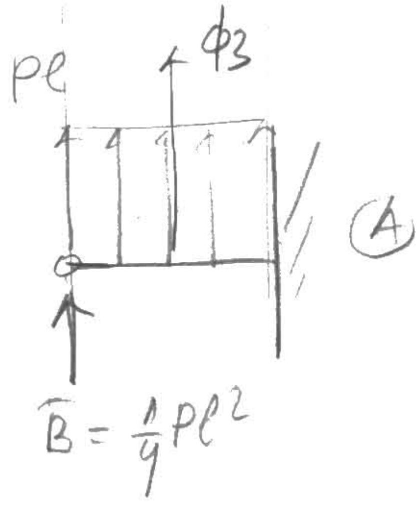




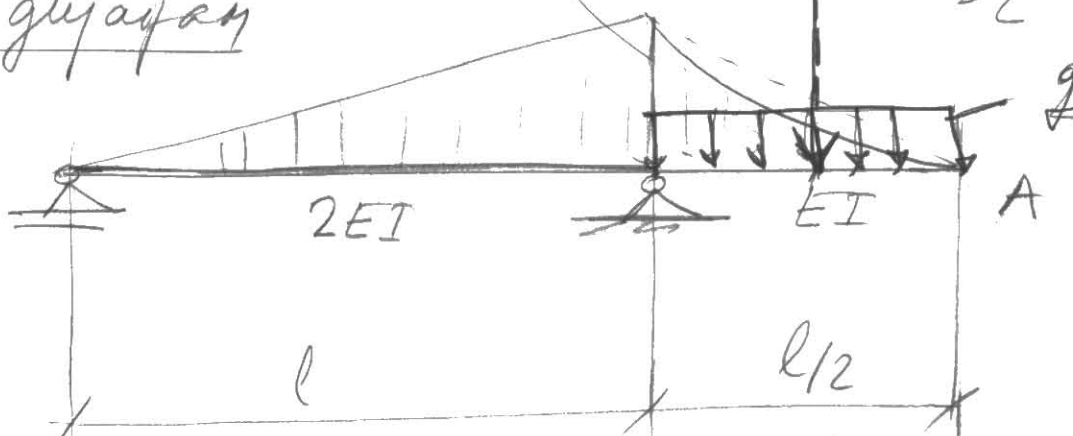
$$\phi_3 = Pl \cdot \frac{l}{2} = \frac{Pl^2}{2}$$

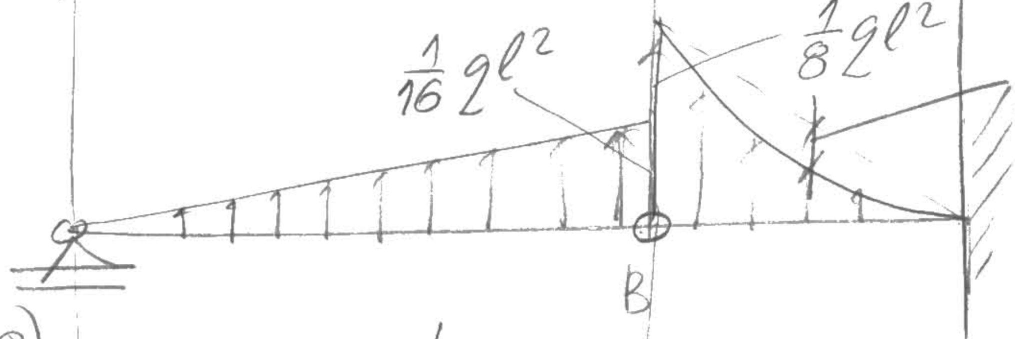
$$w_A = \frac{1}{EI}\left[\frac{1}{4}Pl^2 \cdot \frac{l}{2} + \frac{Pl^2}{2} \cdot \frac{l}{4}\right]$$

$$\varphi_A = \frac{1}{EI}\left[\frac{1}{4}Pl^2 + \frac{Pl^2}{2}\right]$$

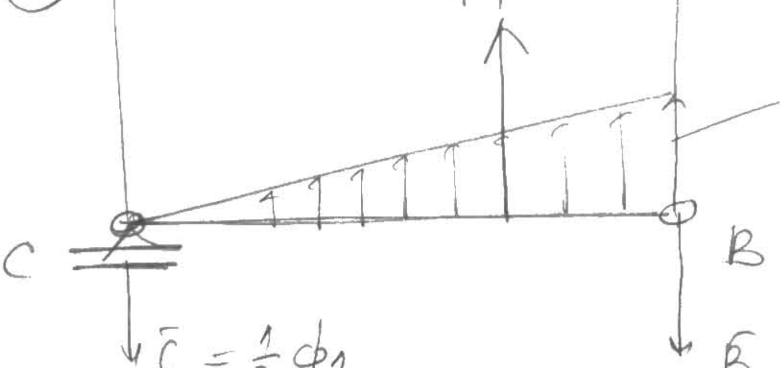


M дијаграм





$f = \frac{1}{8} q \left(\frac{l}{2}\right)^2 = \frac{1}{32} q l^2$



$\Phi_1 = \frac{1}{2} l \cdot \frac{1}{16} q l^2$

$\Phi_1 = \frac{1}{32} q l^3$

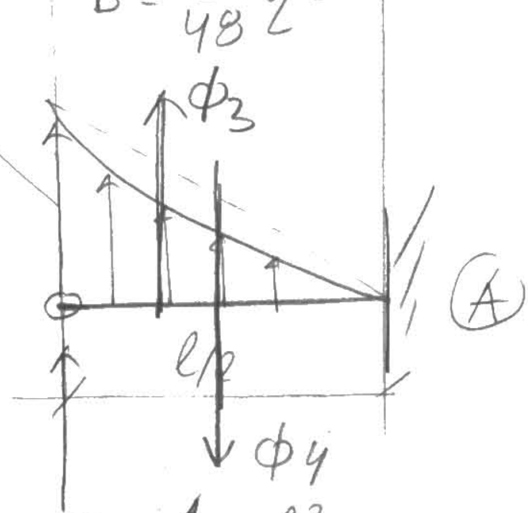
$\bar{C} = \frac{1}{3} \Phi_1$

$\bar{B} = \frac{2}{3} \Phi_1 = \frac{2}{3} \cdot \frac{1}{32} q l^3$

$\bar{B} = \frac{1}{48} q l^3$

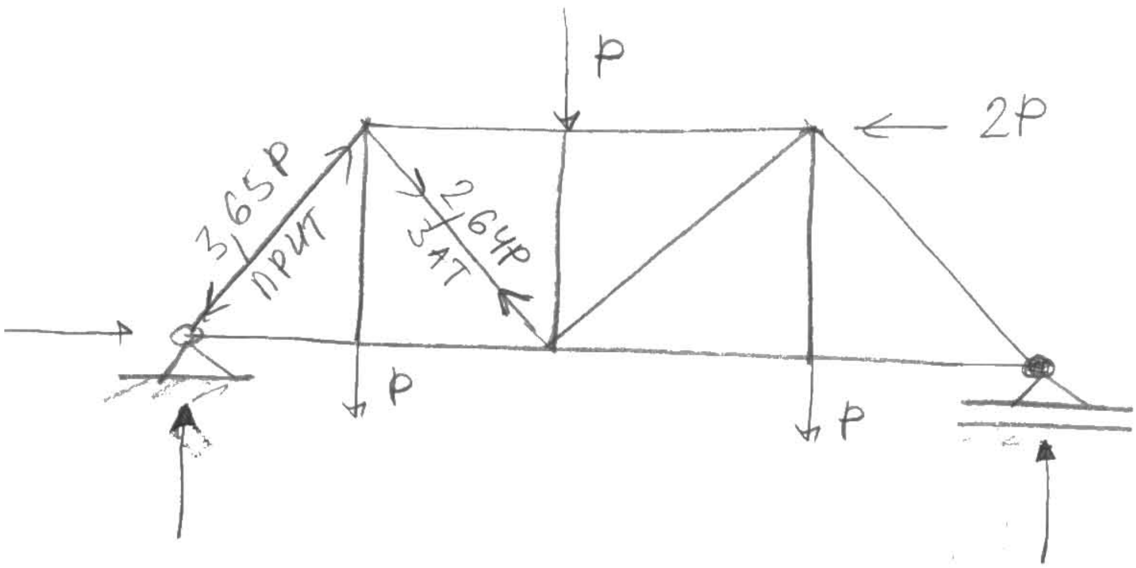
$\Phi_3 = \frac{1}{2} \frac{l}{2} \cdot \frac{1}{8} q l^2 = \frac{1}{32} q l^3$

$\Phi_4 = \frac{2}{3} l f = \frac{2}{3} \cdot \frac{l}{2} \cdot \frac{1}{32} q l^2 = \frac{1}{96} q l^3$



$\bar{B} = \frac{1}{48} q l^3$

Извијање



Одредити параметар P тако да код затегнутих штапова коеф сигурности на лом буде $n_L \geq 2{,}0$; а код притиснутих штапова ($n_S \geq 3{,}0$) коеф сигурности на извијање буде већи од 3.

затегнути штапови

$S = 2{,}64P$ $\quad \boxed{N^* = A \cdot \sigma_T}$ — максимална носивост штапа пред лом.

$$n_L = \frac{N^*}{S} \geq 2{,}0$$

притиснути штапови

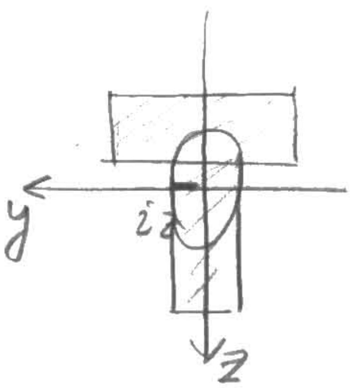
$S = 3{,}65P$ $\quad \boxed{N^* = A \cdot \sigma_{MIN}}$

$$n_S = \frac{N^*}{S} \geq 3{,}0$$

$$\sigma_{min} = \min \begin{cases} \sigma_E = \dfrac{\pi^2 \cdot E}{\lambda^2} \; [MPa] \\ \sigma_{KT}^{T} = a - b\lambda \; [MPa] \\ \sigma_T = 240 MPa \; [MPa] \end{cases}$$

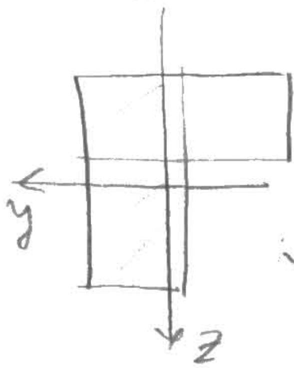
λ = витkост штапа [без димензије]

$$\lambda = \frac{l_0 \, [cm]}{i_{min} \, [cm]}$$



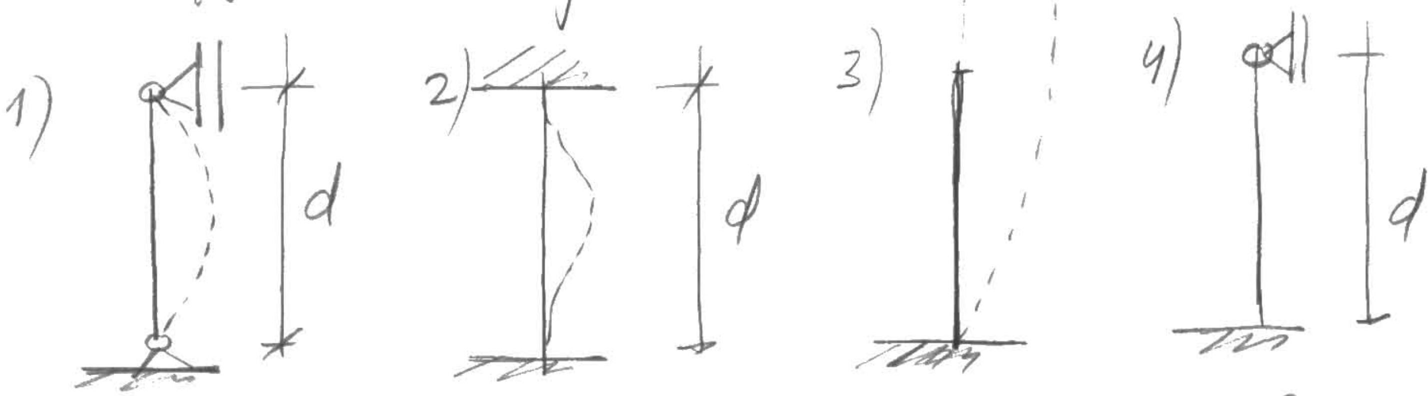
$$i_{min} = \min \begin{cases} i_y \\ i_z \end{cases}$$

$$i_y = \sqrt{\frac{I_y}{A}} \quad i_z = \sqrt{\frac{I_z}{A}}$$



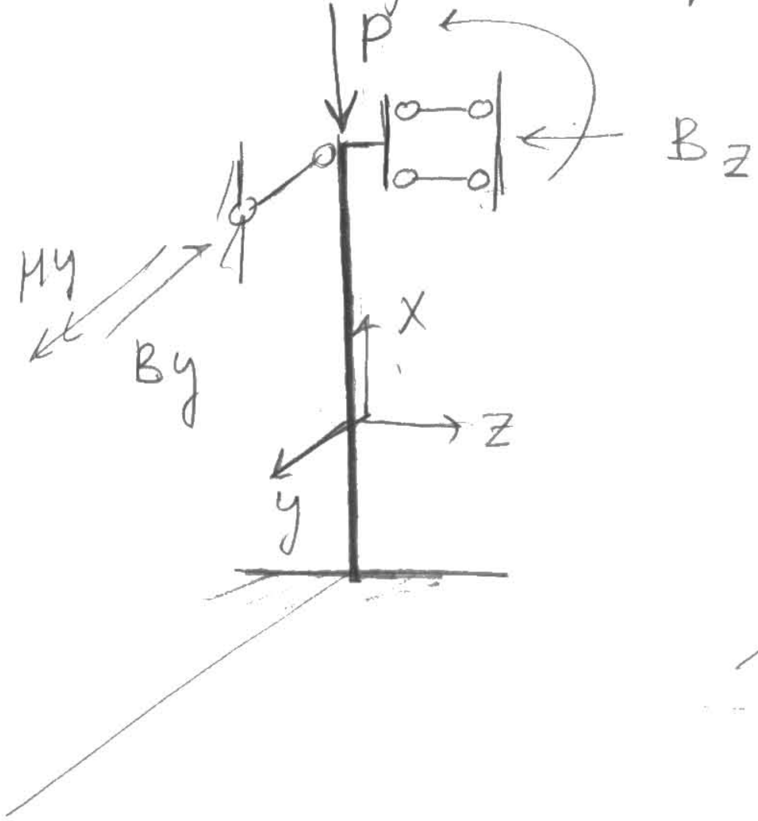
$$i_{min} = i_z = \sqrt{\frac{I_z}{A}}$$

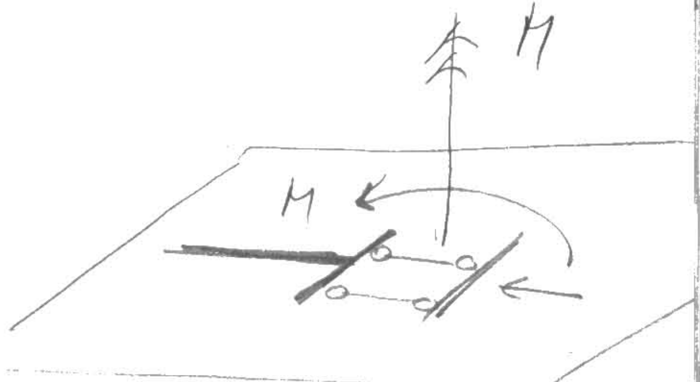
l_0 = дужина извијања

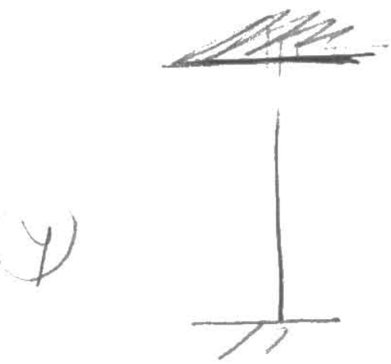


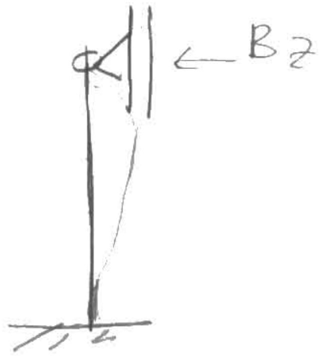
$l_0 = d$ $\qquad$ $l_0 = \frac{1}{2} d$ $\qquad$ $l_0 = 2d$ $\qquad$ $l_0 = \frac{\sqrt{2}}{2} d$

Извијање у простору



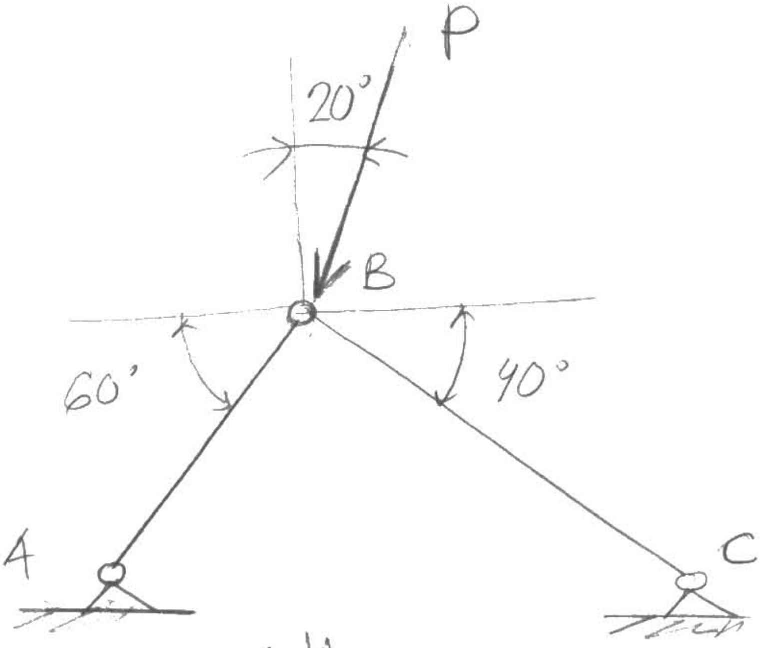






$l_{oy} = \frac{d}{2}$

$l_{oz} = \frac{\sqrt{2}}{2} d$



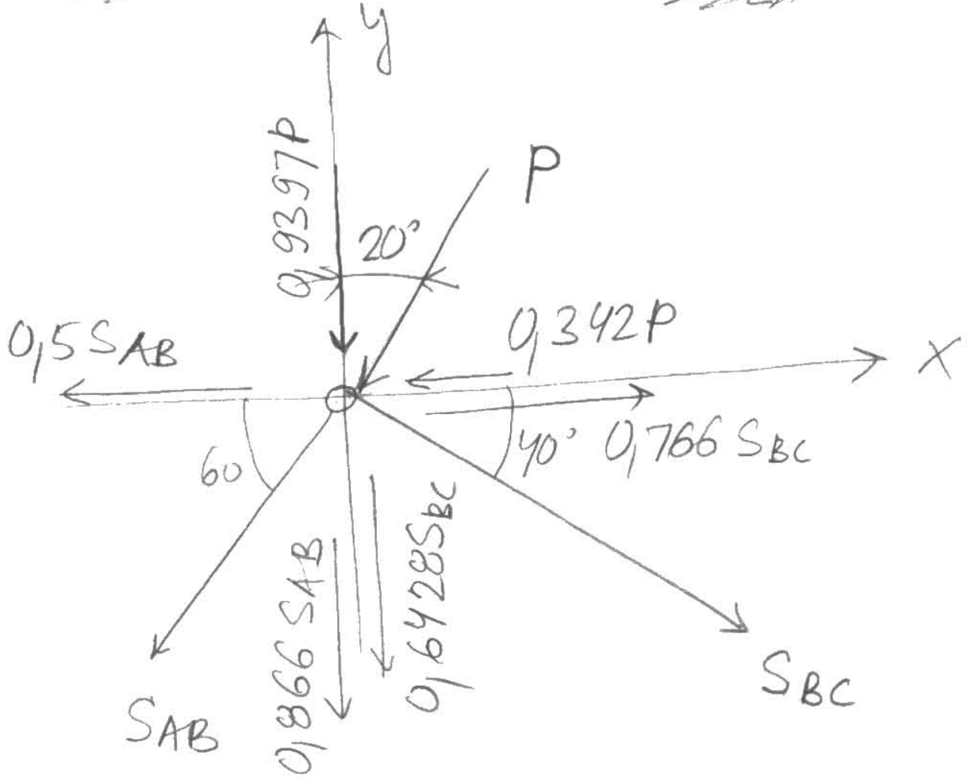
$l_{AB} = 3\,m$

$l_{BC} = 3{,}5\,m$

$E = 210\ GPa$

$\sigma_T = 240\ MPa$

$\sigma_{KR}^{T} = 310 - 1{,}92\lambda\ [MPa]$



(1) $\Sigma X = 0$

$-0{,}5\,S_{AB} + 0{,}766\,S_{BC} - 0{,}342\,P = 0$

(2) $\Sigma Y = 0$

$-0{,}866\,S_{AB} - 0{,}6428\,S_{BC} - 0{,}9397\,P = 0$

$-0{,}5\,S_1 + 0{,}766\,S_2 = 0{,}342\,P$

$-0{,}866\,S_1 - 0{,}6428\,S_2 = 0{,}9397\,P$

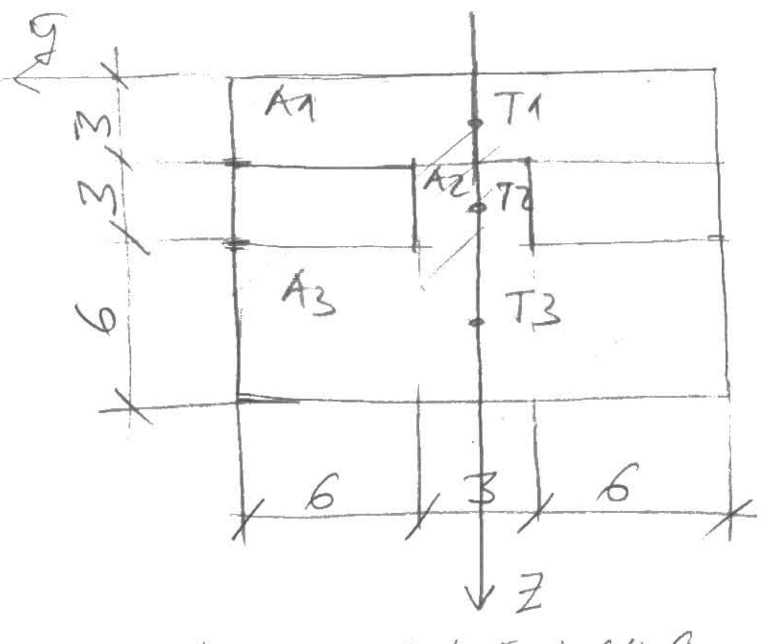
$$D = \begin{vmatrix} -0{,}5 & 0{,}766 \\ -0{,}866 & -0{,}6428 \end{vmatrix} = \begin{matrix} -0{,}5\cdot(-0{,}6428) - 0{,}766\cdot(-0{,}866) \\ \underline{\underline{0{,}9848}} \end{matrix}$$

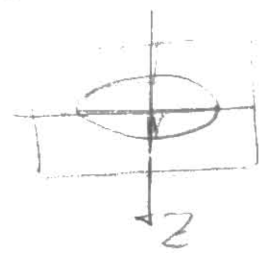
$$D_{S_1} = \begin{vmatrix} 0{,}342P & 0{,}766 \\ 0{,}9397P & -0{,}6428 \end{vmatrix} \begin{matrix} 0{,}342P\cdot(-0{,}6428) - 0{,}766\cdot 0{,}9397P \\ = -0{,}9396P \end{matrix}$$

$$D_{S_2} = \begin{vmatrix} -0{,}5 & 0{,}342P \\ -0{,}866 & 0{,}9397P \end{vmatrix} \begin{matrix} -0{,}5\cdot 0{,}9397P - 0{,}342P\cdot(-0{,}866) \\ = \underline{-0{,}1737P} \end{matrix}$$

$$S_1 = S_{AB} = \frac{D_{S1}}{D} = \frac{-0{,}9396P}{0{,}9848} = \underline{\underline{-0{,}954P}}$$

$$S_2 = S_{BC} = \frac{D_{S2}}{D} = \frac{-0{,}1737P}{0{,}9848} = \underline{\underline{-0{,}176P}}$$





$A_1 = 45 \quad \cancel{T_1(0;1,5)}$

$A_2 = 9 \quad \cancel{T_2(0;4,5)}$

$A_3 = 90 \quad \cancel{T_3(0;9)}$

$A = 144 \quad T_1(0;-4{,}875)$

$T_2(0;-1{,}875)$

$T_3(0;2{,}625)$

$$z_T = \frac{45\cdot 1{,}5 + 9\cdot 4{,}5 + 90\cdot 9}{144} = 6{,}375$$

$$I_y = \frac{15\cdot 3^3}{12} + 45\cdot(-4,875)^2 + \frac{3^4}{12} + 9\cdot(-1,875)^2 +$$

$$+ \frac{15\cdot 6^3}{12} + 90\cdot 2,625^2 = \underline{2031,75\ cm^4}$$

$$I_z = \frac{3\cdot 15^3}{12} + \frac{3^4}{12} + \frac{6\cdot 15^3}{12} = 2538\ cm^4$$

$$i_{min} = i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2031,75\ cm^4}{144}} = \underline{3,756\ cm}$$

<u>штап AB</u> $\quad \underline{S_1 = 0,954P}$

$$l_0 = d = 300\ cm$$

$$\lambda = \frac{l_0}{i_{min}} = \frac{300\ cm}{3,756} = 79,87$$

$$\sigma_{MIN} = min \begin{cases} \sigma_E = \frac{\pi^2\cdot E}{\lambda^2} = \frac{\pi^2\cdot 210\,000\ MPa}{79,87^2} = 324,9\ MPa \\ \sigma_i^{TET} = 310 - 1,92\cdot 79,87 = \underline{156,64\ MPa} \\ \sigma_T = 240\ MPa \end{cases}$$

$$N^* = A\,\sigma_{MIN} = 144\ cm^2\cdot 15,664\ \frac{kN}{cm^2} = 2255\ kN$$

$$n_S = \frac{N^*}{S_{AB}} \geq 3,5 \qquad \frac{2255\ kN}{0,954P} \geq 3,5$$

$$\boxed{P \leq 675,35\ kN}$$

$$\lambda = \frac{350\ cm}{3,756} = 93,18$$

<u>штап BC</u> $\quad S_2 = 0,176P$

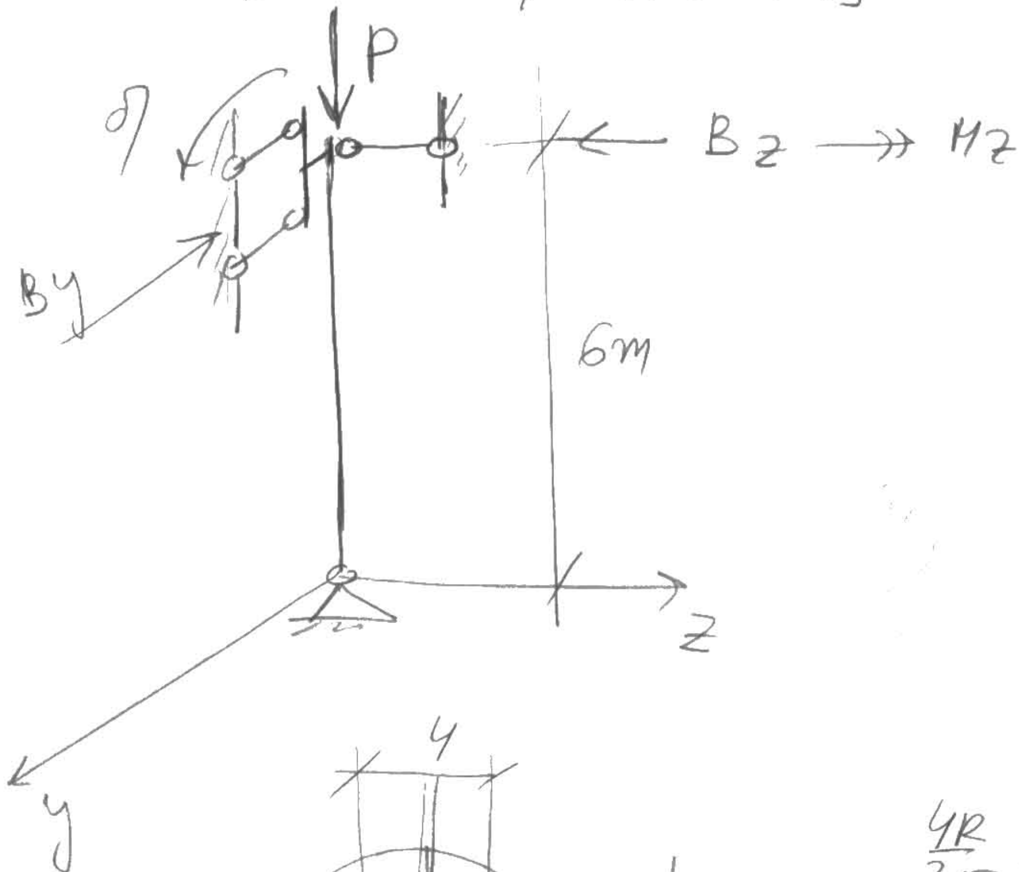
$$\sigma_{MIN} = min \begin{cases} \sigma_E = \frac{\pi^2\cdot 210000\ MPa}{93,18^2} = 238,71\ MPa \\ \sigma_{KR}^{T} = 310 - 1,92\cdot 93,18 = \underline{131,10\ MPa} \end{cases}$$

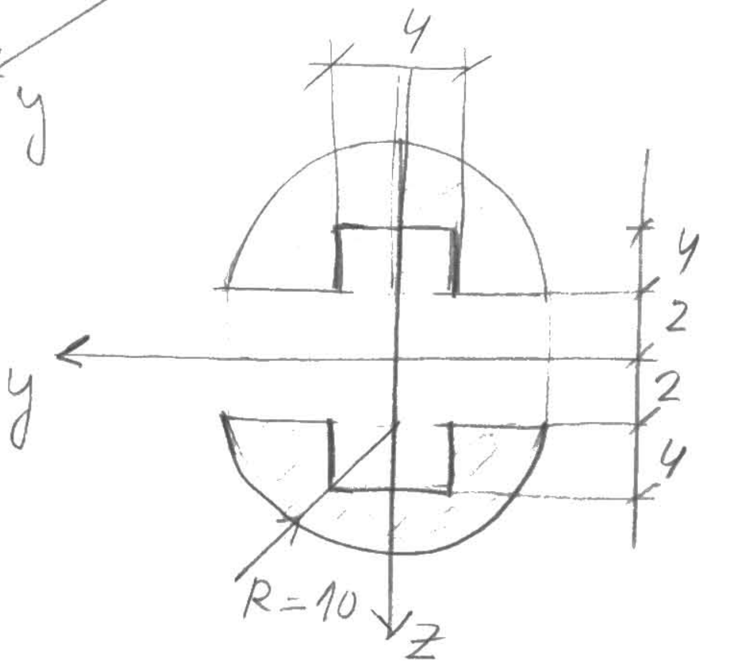
$$N^* = 144\ cm^2\cdot 13,11\ \frac{kN}{cm^2} = 1887,84\ kN$$

$$n_S = \frac{1887,84}{0,176P} \geq 3,5 \qquad P \leq 3064,67\ kN$$

$E = 210\,GPa \qquad \sigma_T = 240\,MPa$

$\sigma_{kr}^T = 310 - 1{,}92\lambda\ [MPa]$





$\frac{4R}{3\pi} = \frac{4 \cdot 10}{3\pi} = 4{,}24$

$A_1 = \frac{1}{2} R^2\pi = \frac{1}{2} \cdot 10^2\pi = 157{,}08\,cm^2 \qquad T_1(0; -6{,}24)$

$A_2 = 16 \qquad T_2(0; -4)$

$I_y = 2(0{,}1097 \cdot 10^4 + 157{,}08 \cdot 6{,}24^2) - 2 \cdot \left(\frac{4^4}{12} + 16 \cdot 4^2\right)$

$I_y = \underline{13\,871{,}97\,cm^4}$

$I_z = \frac{10^4\pi}{4} - \frac{8 \cdot 4^3}{12} = \underline{7811{,}31\,cm^4}$

извијање око (y) осе

$l_{oy} = d = 600\,cm$

$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{13\,871{,}97}{282{,}16}} = 7{,}011\,cm$

$\lambda_y = \frac{l_{oy}}{i_y} = \frac{600\,cm}{7{,}011\,cm} = 85{,}58$

$$\sigma_{MIN} = \min \begin{cases} \sigma_E = \frac{\pi^2 \cdot 210\,000}{85{,}58^2}\,MPa = 282{,}9\,MPa \\ \sigma_{KR}^T = 310 - 1{,}92 \cdot 85{,}58 = \underline{145{,}68\,MPa} \\ \sigma_T = 240\,MPa \end{cases}$$

$N^* = A\,\sigma_{MIN} = 282{,}16\,cm^2 \cdot 14{,}568\,\frac{kN}{cm^2} = 4110{,}5\,kN$

$P_{KRy} = 4110{,}5\,kN$

извијање око z осе

$l_{oz} = \frac{\sqrt{2}}{2} d = \frac{\sqrt{2}}{2} \cdot 600\,cm$

$l_{oz} = 424{,}26\,cm$

$\lambda_z = \frac{l_{oz}}{i_z} = \frac{424{,}26\,cm}{5{,}26\,cm}$

$i_z = \sqrt{\frac{7811{,}31}{282{,}16}} = 5{,}26\,cm$

$\underline{\lambda_z = 80{,}66}$

$$\sigma_{MIN} = \min \begin{cases} \sigma_E = \frac{\pi^2 \cdot 210\,000}{80{,}66^2} = \cancel{101{,}4\,MPa}\ 318{,}57 \\ \sigma_{KR}^T = 310 - 1{,}92 \cdot 80{,}66 = 155{,}13\,MPa \\ \sigma_T = 240\,MPa \end{cases}$$

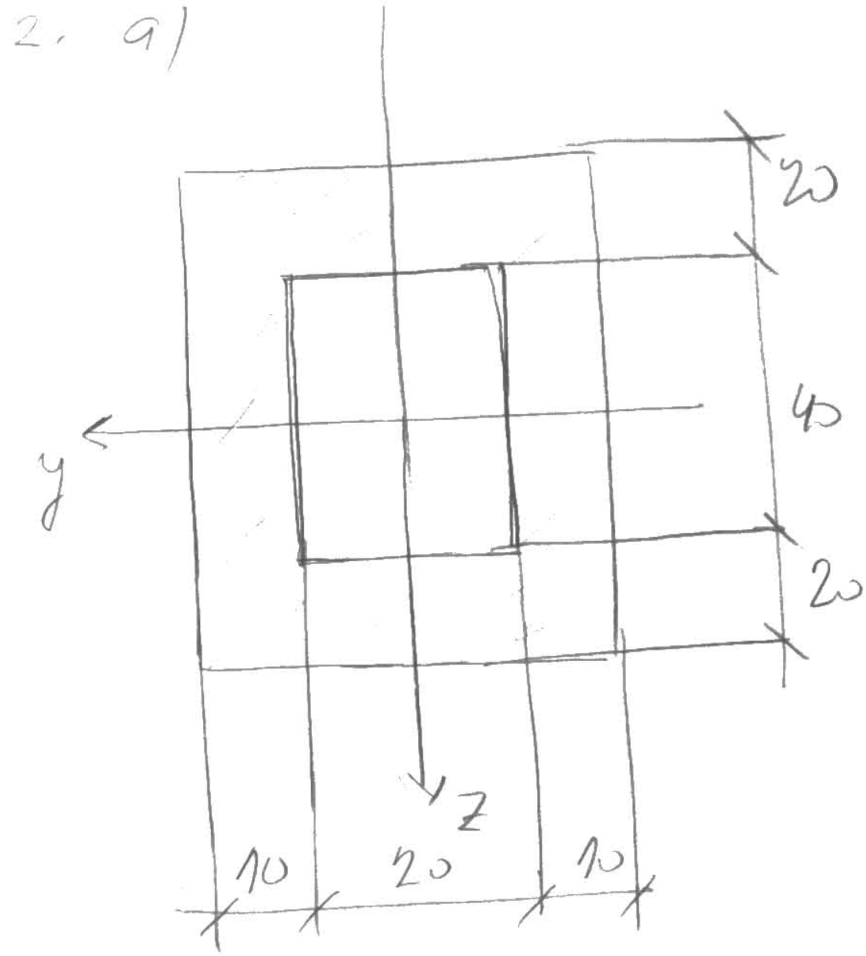
$N^* = A\,\sigma_{MIN} = 282{,}16 \cdot 10{,}14\,\frac{kN}{cm^2} = 2861{,}1\,kN$

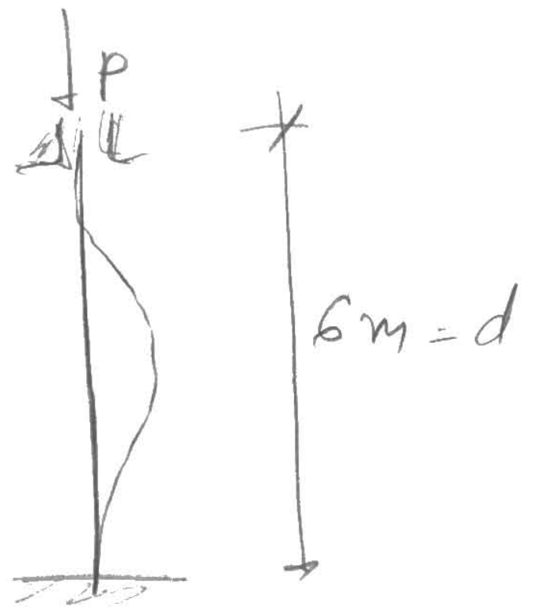
$\boxed{P_{KRz} = 2861{,}1\,kN}$ ← X

НИЈЕ ТАЧНО

$P_{kr}^z = 4377{,}23\,kN$

2. a)





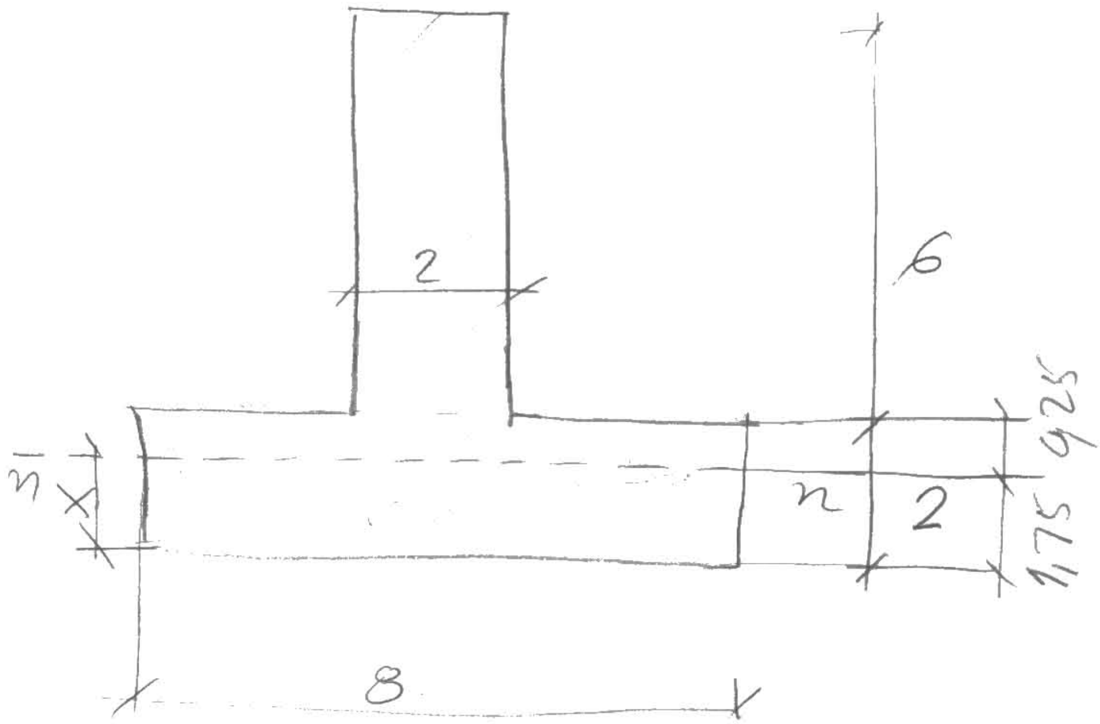
$\underline{l_0 = \frac{d}{2}}$ $\quad \lambda = \underline{2 l_0}$

b) $d = ?$ $\qquad P = 700\,kN$ $\qquad n_S = 2$

$$n_S = \frac{N^*}{P} \geq 2 \qquad N^* = 1400\,kN$$

$$N^* = A \cdot \sigma_{MIN} \Rightarrow \underline{\sigma_{MIN}} = \quad MPa$$

$$\sigma_{MIN} = \begin{cases} \sigma_E = \quad \lambda \rightarrow \lambda = \frac{l_0}{i_{min}} \Rightarrow d \\ \sigma_{KR}^T = \quad \lambda \rightarrow \lambda = \frac{l_0}{i_{min}} \Rightarrow d \\ \sigma_1 \end{cases}$$



$$A = 6 \cdot 2 + 8 \cdot 2 = 28 \qquad \frac{A}{2} = 14$$

$$8 \cdot x = \frac{A}{2} \qquad 8 \cdot x = 14 \qquad x = 1{,}75$$

$$W^* = \sum A \cdot d_{Tn} = 8 \cdot 1{,}75 \cdot \frac{1{,}75}{2} + 8 \cdot 0{,}25 \cdot \frac{0{,}25}{2} +$$

$$+ 6 \cdot 2 \cdot 3{,}25 = 51{,}5 \text{ cm}^3$$

$$M^* = \sigma_T \cdot W^* =$$

$$f = \frac{W^*}{W_y}$$

f – фактор облика

W_y – отпорни моменат

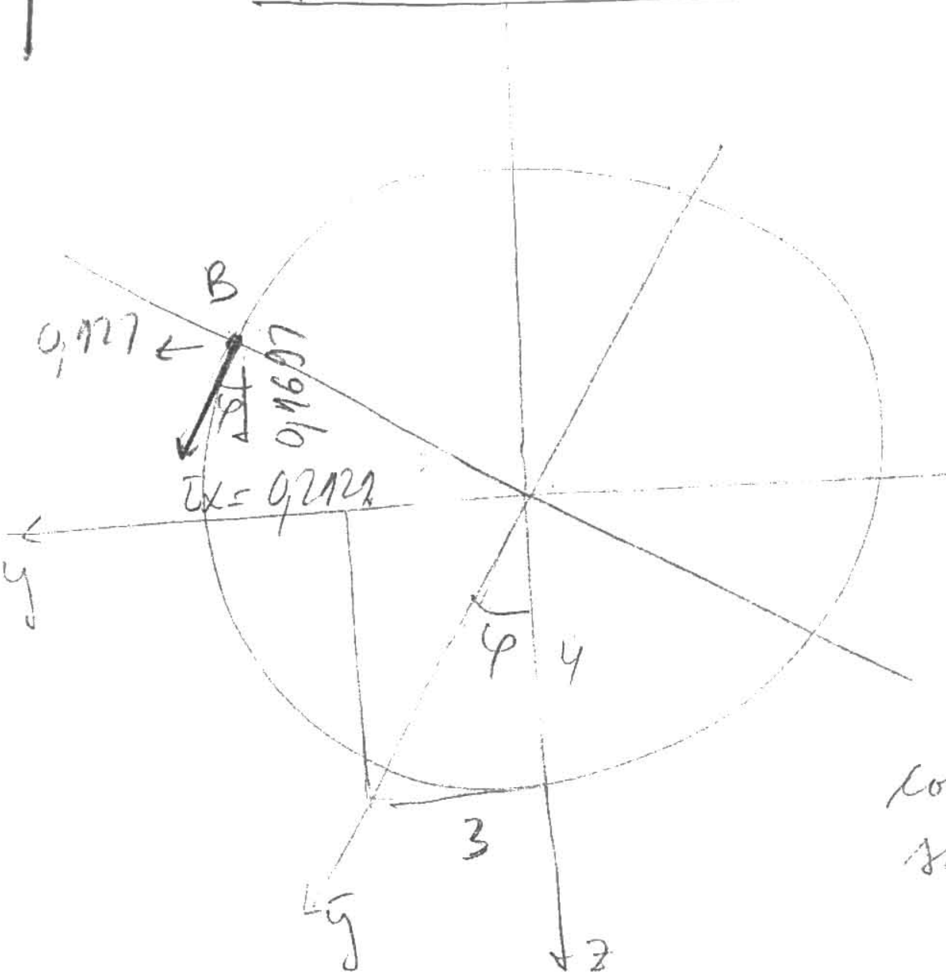
M^* – гранични моменат

W^* – отпорни пластични моменат

$$W_y = \frac{I_y}{z_{max}}$$

ИСПИТНИ ЗАДАЧИ

ИСПИТНИ ЗАДАЧИ



$\cos\varphi = 0{,}8$

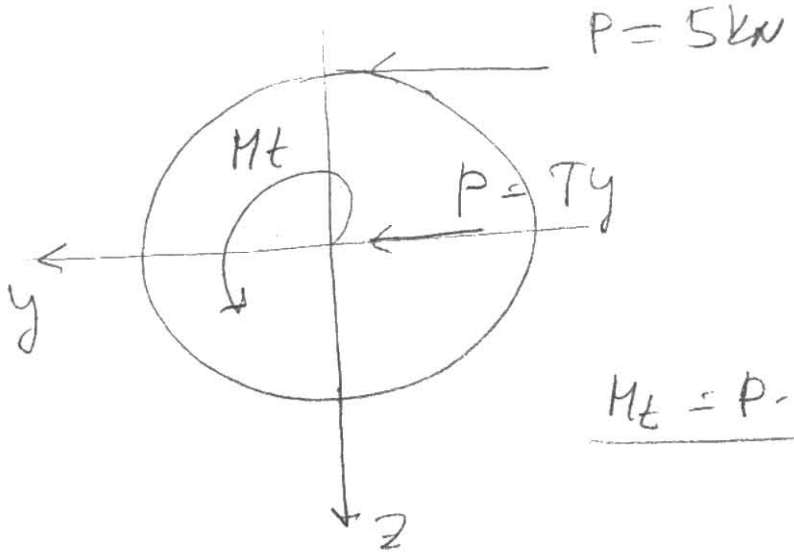
$\sin\varphi = 0{,}6$

$$S_{xyz}^{B} = \begin{bmatrix} 0{,}0796 & 0{,}127 & 0{,}1697 \\ 0{,}127 & 0 & 0 \\ 0{,}1697 & 0 & 0 \end{bmatrix} \text{MPa}$$

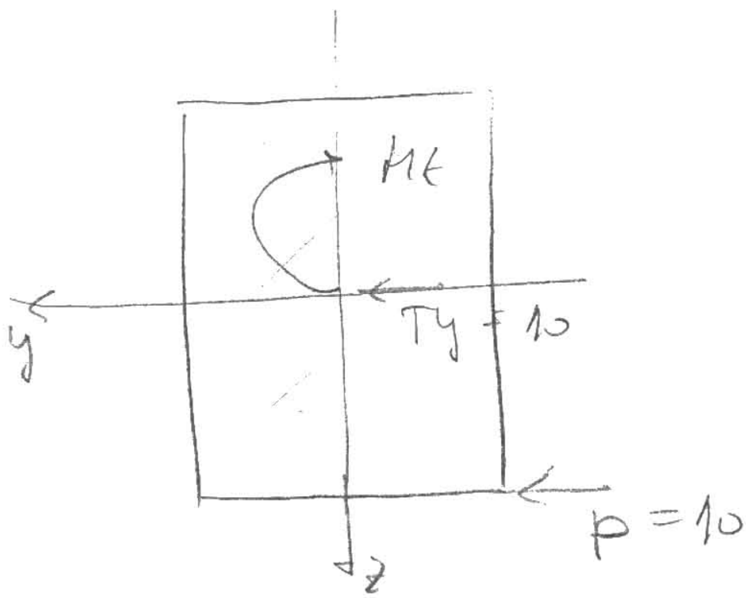
$$\sigma^3 - I_1\sigma^2 + I_2\cdot\sigma - \cancel{I_3}^{\to 0} = 0$$

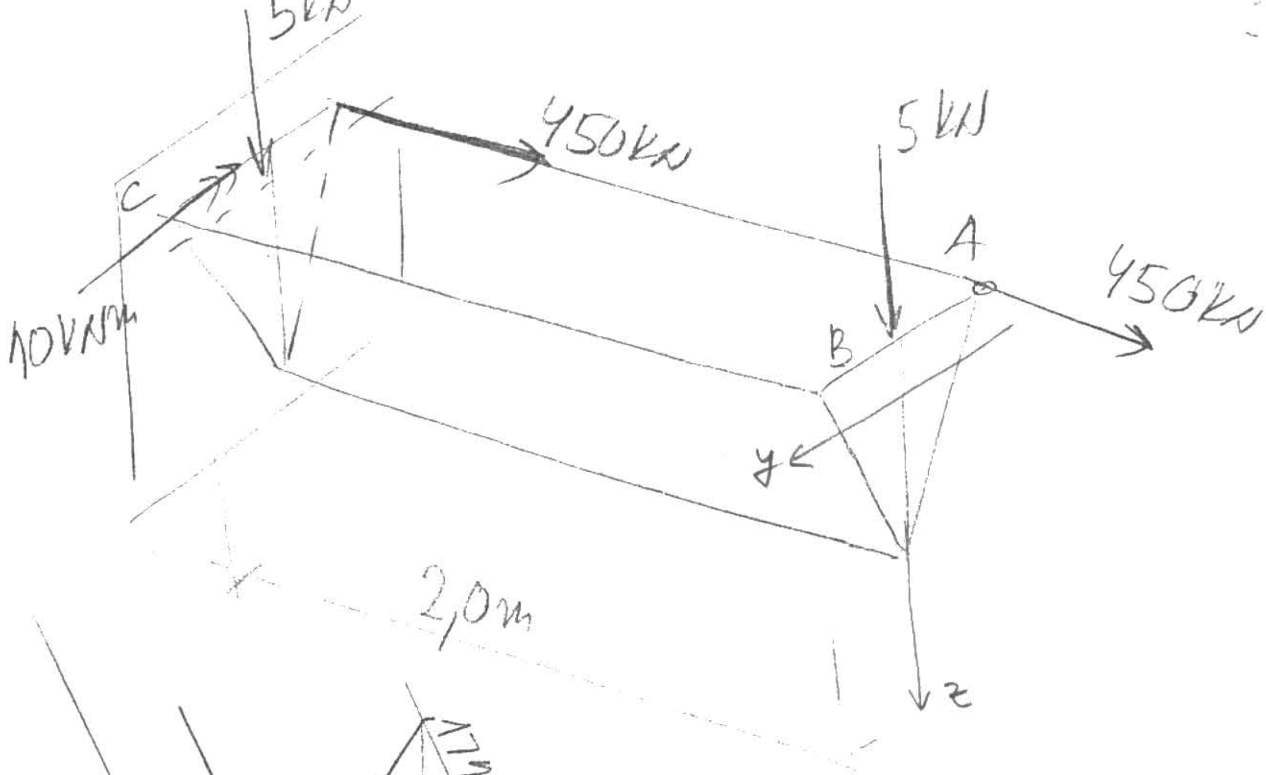
$\sigma_1 \quad \sigma_2$

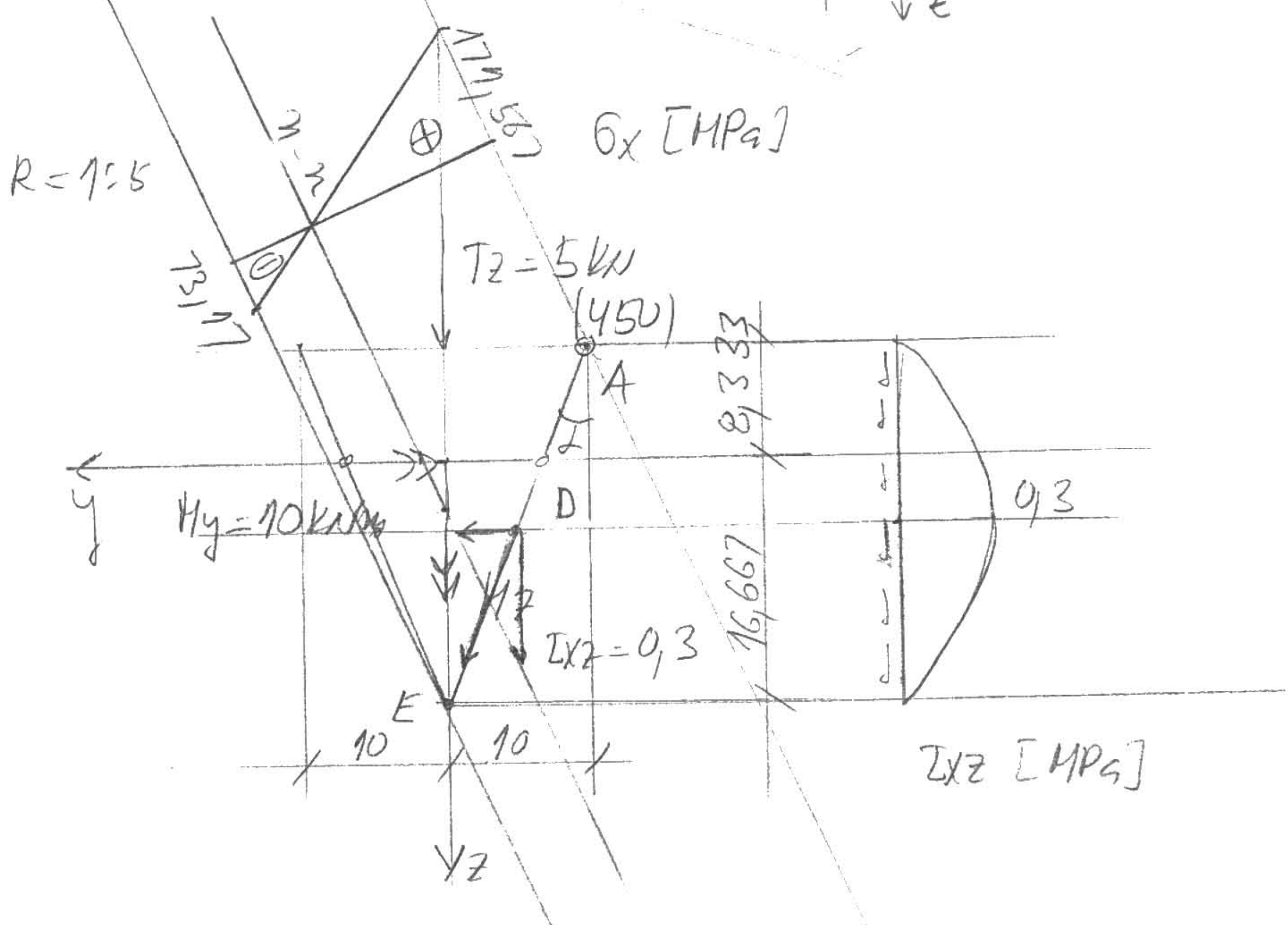
$$\tau_{max} = \frac{|\sigma_1 - \sigma_2|}{2}$$



$M_t = P \cdot R$







$N = 450 kN$

$M_y = 10 kNm + 450 kN \cdot 8{,}333 \cdot 10^{-2} m = 47{,}485 kNm$

$M_z = 45 kNm$

$I_y = \frac{20 \cdot 25^3}{36} = 8680,55\ cm^4$ $\quad$ $A = 250\ cm^2$ $\quad$ 4

$I_z = \frac{25 \cdot 20^3}{48} cm^4 = 4166,66\ cm^4$

$\sigma_x = \frac{450\ kN}{250\ cm^2} - \frac{4748,5\ kNcm}{8680,55} \cdot z - \frac{4500\ kNcm}{4166,66} y$

n-n linija

$\sigma_x = 0$

$\frac{450}{250} - \frac{4748,5}{8680,55} z - \frac{4500}{4166,66} y = 0$

y	0	1,67
z	3,27	0

$A(-10; -8,333)$

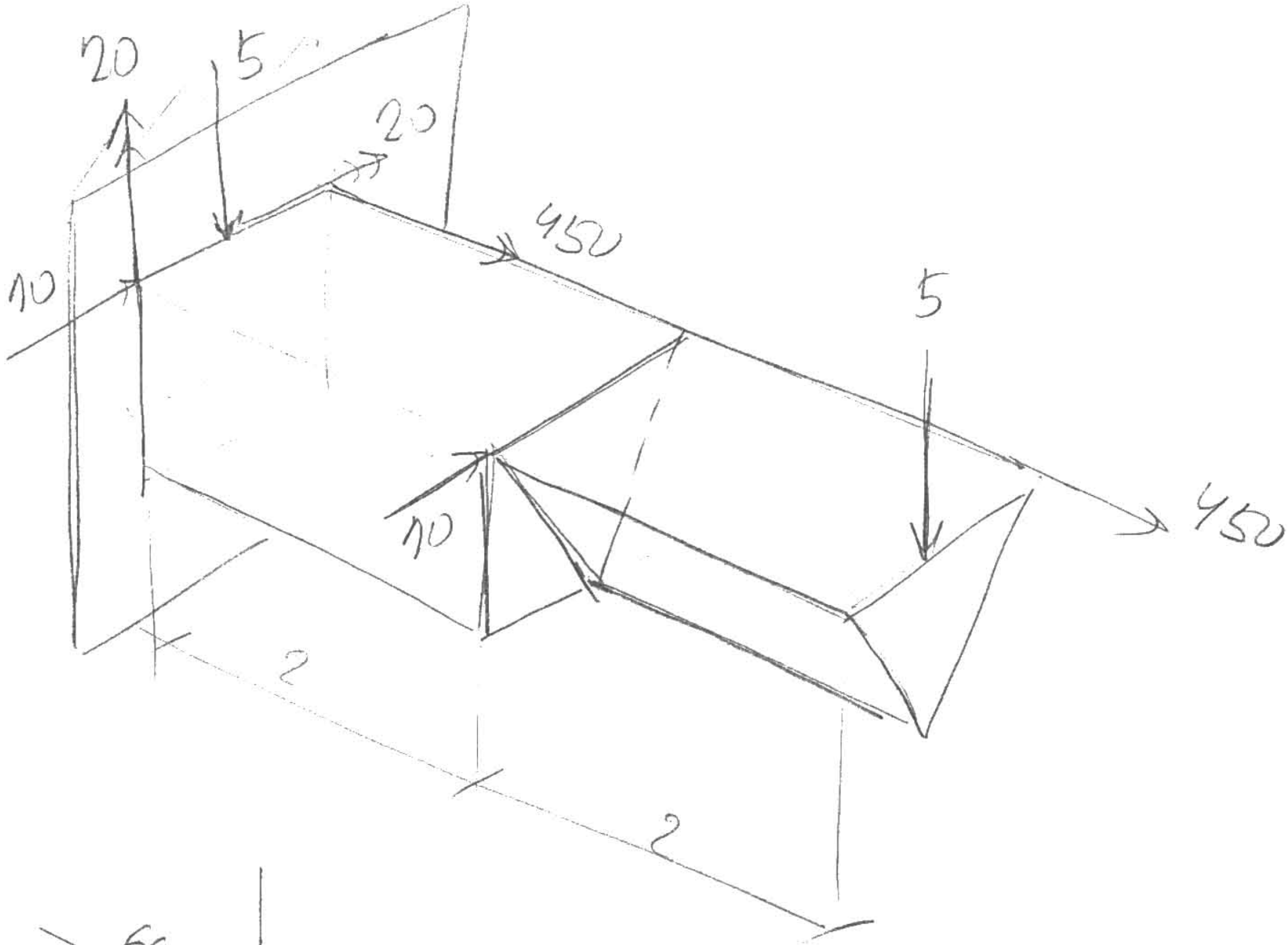
$E(0; 16,667)$

$\sigma_x^A = \frac{450}{250} - \frac{4748,5}{8680,55} \cdot (-8,333) - \frac{4500}{4166,67} \cdot (-10)$

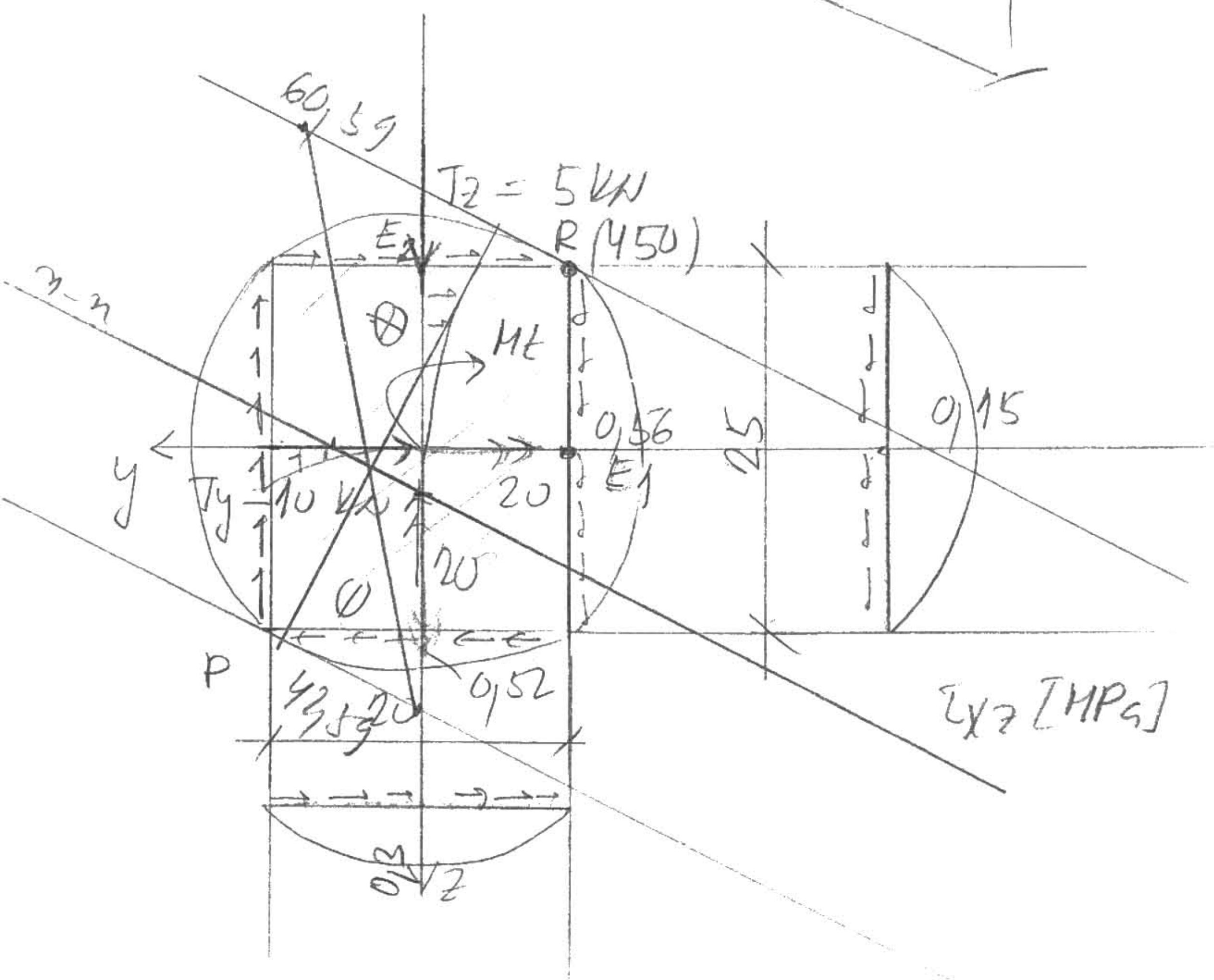
$\sigma_x^A = 171,567\ MPa$

$\sigma_x^E = \frac{450}{250} - \frac{4748,5}{8680,55} \cdot 16,667 = -73,17\ MPa$

$\tau_x = \frac{3}{2} \cdot \frac{5\ kN}{250\ cm^2} = 0,30\ MPa$ ✓



20
5
20
450
10
5
10
450
2
2



60,59
Tz = 5 kN
E
R (450)
Mt
0,56
0,15
25
y
Ty = 10 kN
20
E1
20
P
42,59
20
0,52
τxz [MPa]
n-n
0,3
z

$M_t = 10kN \cdot 12{,}5\,cm = 125\,kNcm$

$T_y = 10\,kN$

$T_z = 5\,kN$

$N = 450$

$M_y = 2000\,kNcm + 450 \cdot 12{,}5 = 7625\,kNcm$

$M_z = -2000\,kNcm + 450 \cdot 10 = 2500\,kNcm$

$\tau_x^{T_y} = \frac{3}{2} \cdot \frac{10}{500} = 0{,}3\,MPa$

$\tau_x^{T_z} = \frac{3}{2}\,\frac{5\,kN}{500\,cm^2} = 0{,}15\,MPa$

$\mu = \frac{c}{b} = 1{,}25$

$\beta = 0{,}2195 \qquad \gamma = 0{,}239$

$W_{tB} = 0{,}2195 \cdot 20^2 \cdot 25 = 2195$

$W_{tC} = 0{,}239 \cdot 20^2 \cdot 25 = 2390$

$\tau_{xB} = \frac{125\,kNcm}{2195} = 0{,}56\,MPa$

$\tau_{xC} = \frac{125\,kNcm}{2390} = 0{,}52\,MPa$

$A = 500\,cm^2$

$$I_y = \frac{20 \cdot 25^3}{12} = 26\,041{,}67$$

$$I_z = \frac{25 \cdot 20^3}{12} = 16\,666{,}67\,cm^4$$

$$\sigma_x = \frac{450}{500} - \frac{7625}{26\,041{,}67} \cdot z - \frac{2\,500}{16\,666{,}67}\, y$$

$\underline{n-n}$

$\sigma_x = 0$

$$\frac{450}{500} - \frac{7625}{26\,041{,}67} \cdot z - \frac{2\,500}{16\,666{,}67}\, y = 0$$

y	0	6,0
z	3,07	0

$$\sigma_x^{R} = \frac{450}{500} - \frac{7625}{26\,041{,}67} \cdot (-12{,}5) - \frac{2\,500}{16\,666{,}67} \cdot (-10) = 60{,}59\,MPa$$

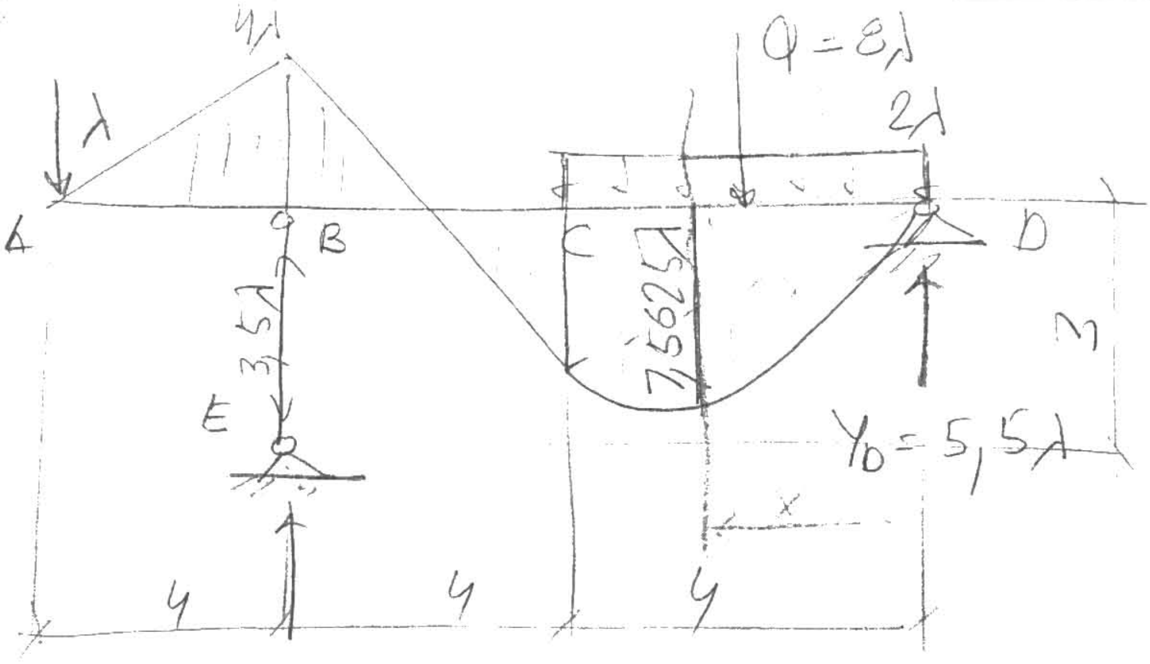
$$\sigma_x^{P} = \frac{450}{500} - \frac{7625}{26\,041{,}67} \cdot (12{,}5) - \frac{2\,500}{16\,666{,}67} \cdot 10 = -42{,}59\,MPa$$

$$\tau_x^{E1} = 0{,}56 + 0{,}15 = 0{,}71\,MPa$$

$$\tau_x^{E2} = 0{,}3 + 0{,}52 = \underline{0{,}82\,MPa}$$

$$\sigma_x^{E2} = \frac{450}{500} - \frac{7625}{26\,041{,}67}(-13{,}0) = 45{,}59\,MPa$$

$$S_{E2} = \begin{bmatrix} 45{,}59 & -0{,}82 & 0 \\ -0{,}82 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$



$Y_E = 3,5\lambda$

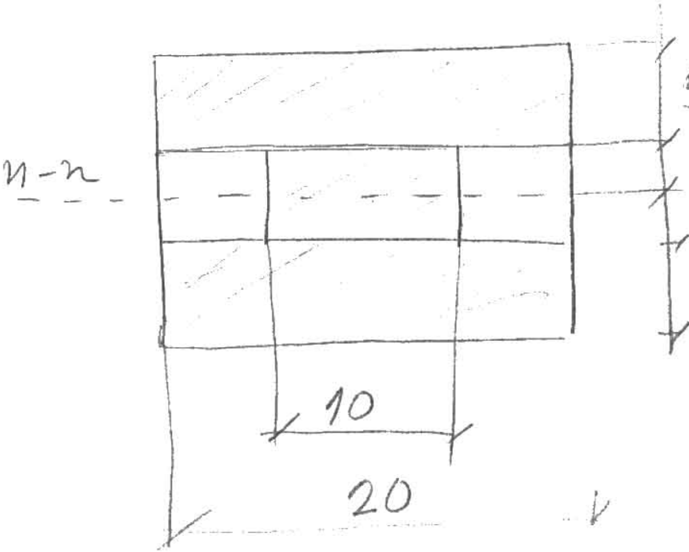
(1) $\overset{\curvearrowright}{\Sigma M_E} = 0 \qquad -\lambda\cdot 4 + 8\lambda\cdot 6 - Y_D\cdot 8 = 0$

(2) $\overset{\curvearrowright}{\Sigma M_D} = 0 \qquad Y_E\cdot 8 - \lambda\cdot 12 - 8\lambda\cdot 2 = 0$

$T(x) = 0 \qquad -5,5\lambda + 2\lambda\cdot x = 0 \qquad x = 2,75\ m$

$\max M = 5,5\lambda\cdot x - (2\lambda\cdot x)\frac{x}{2} = 7,5625\lambda$

A – B – C – D



$W^y = (20\cdot 5\cdot 5 + 10\cdot 2,5\cdot 1,25)$

$W^y = 1062,5\ cm^3$

$M^y = W^y\cdot \sigma_T$

$M^y = 24\ \frac{kN}{cm^2}\cdot 1062,5\ cm^3$

$M^y = 255\ kNm$

$$\sigma_{MIN} = \begin{cases} \sigma_E = \dfrac{\pi^2 \cdot 210000}{208{,}33^2} = \underline{47{,}75\ MPa} \\ \sigma_{kr}^{T} = 300 - 1{,}2 \cdot 208{,}33 = 50\ MPa \end{cases}$$

$$\sigma_T = 240\ MPa$$

$$N^d = 35\ cm^2 \cdot 4{,}775\ \frac{kN}{cm^2} = 167{,}125\ kN$$

$$S = 3{,}5\lambda = 167{,}125\ kN$$

$$\underline{\lambda = 47{,}74}$$